Summer Assignment
AP Calculus BC

Directions: Show all work for the following problems. If you had non-AP calculus last year, there may be a couple of parts of the FRQ at the end of the assignment that you may not know how to do. See if you can find some assistance with online resources or email your previous teacher for help.

Academic Integrity: All work should be completed independently and without the assistance of unapproved resources. Any work violating academic integrity will be subject to a "0" and any additional consequences as outlined in the Knox Academic Integrity Policy attached to this assignment.

Due Date: Your work is due the first day of your AP Calculus BC class. All late work will be subjected to a grade reduction or penalty as outlined in the course syllabus and copied below:

All major assignments not submitted on the due date will face a 10% deduction of max points per day for up to five (5) days and up to a 50% deduction.
Summer Assignments for AP Classes that are not submitted on time will result in the student being dropped from the course.

If you have any questions or concerns regarding this assignment, please contact the Dean of Academics, Mrs. Pergola, at dpergola@knoxschool.org .

The next few pages contain properties and rules that you should know off the top of your head. I included them for your reference in case you need a refresher.

Properties of Exponents and Logarithms

Exponents

Let a and b be real numbers and m and n be integers. Then the following properties of exponents hold, provided that all of the expressions appearing in a particular equation are defined.

1. $a^m a^n = a^{m+n}$
2. $(a^m)^n = a^{mn}$
3. $(ab)^m = a^m b^m$
4. $\dfrac{a^m}{a^n} = a^{m-n},\ a \neq 0$
5. $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m},\ b \neq 0$
6. $a^{-m} = \dfrac{1}{a^m},\ a \neq 0$
7. $a^{\frac{1}{n}} = \sqrt[n]{a}$
8. $a^0 = 1,\ a \neq 0$
9. $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

where m and n are integers in properties 7 and 9.

Properties of Logarithms (Recall that logs are only defined for positive values of x.)

For the natural logarithm	For logarithms base a
1. $\ln xy = \ln x + \ln y$	1. $\log_a xy = \log_a x + \log_a y$
2. $\ln \dfrac{x}{y} = \ln x - \ln y$	2. $\log_a \dfrac{x}{y} = \log_a x - \log_a y$
3. $\ln x^y = y \cdot \ln x$	3. $\log_a x^y = y \cdot \log_a x$
4. $\ln e^x = x$	4. $\log_a a^x = x$
5. $e^{\ln x} = x$	5. $a^{\log_a x} = x$

Useful Identities for Logarithms

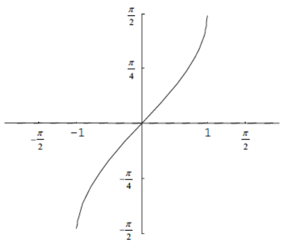
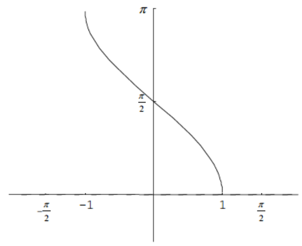
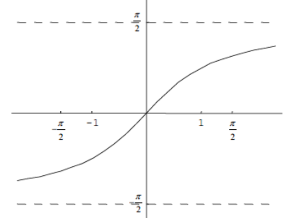
For the natural logarithm	For logarithms base a
1. $\ln e = 1$	1. $\log_a a = 1$, for all $a > 0$
2. $\ln 1 = 0$	2. $\log_a 1 = 0$, for all $a > 0$

If you need a thorough review of logarithms and exponentials, use this link below. The video is great and goes through everything you need to know.

logs and exponentials

Please be familiar with these graphs. More so, the domain and range of the graphs.

GRAPHS OF INVERSE TRIG FUNCTIONS

Domain: $[-1,1]$ Range: $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$  $f(x)=\sin^{-1}x$ $f(x)=\arcsin x$	Domain: $[-1,1]$ Range: $[0,\pi]$  $f(x)=\cos^{-1}x$ $f(x)=\arccos x$	Domain: $(-\infty,\infty)$ Range: $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$  $f(x)=\tan^{-1}x$ $f(x)=\arctan x$

In addition to the above material, you also know the 3 Pythagorean identities, the reciprocal trig identities, and all trig values of special angles (unit circle).

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

Ratio Identities

$tan\theta = \frac{sin\theta}{cos\theta}$

$cot\theta = \frac{cos\theta}{sin\theta}$

Reciprocal Identities

$sin\theta = \frac{1}{csc\theta}$ $\quad cos\theta = \frac{1}{sec\theta}$ $\quad tan\theta = \frac{1}{cot\theta}$

$csc\theta = \frac{1}{sin\theta}$ $\quad sec\theta = \frac{1}{cos\theta}$ $\quad cot\theta = \frac{1}{tan\theta}$

Double Angle Identities

$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$

$\cos(2\theta) = 2\cos^2(\theta) - 1$

$\qquad = 1 - 2\sin^2(2\theta)$

Even/Odd Identities

$\sin(-\theta) = -\sin(\theta)$ $\quad \tan(-\theta) = -\tan(\theta)$ $\quad \cos(-\theta) = \cos(\theta)$

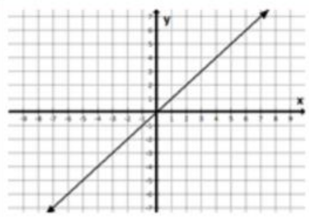
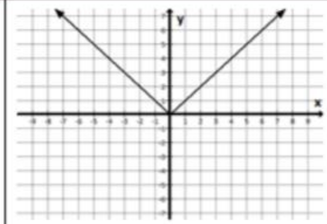
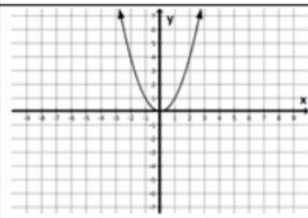
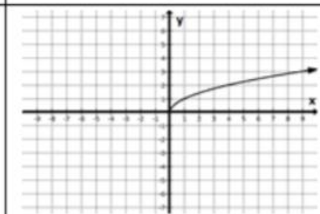
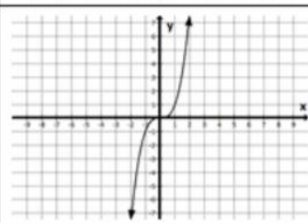
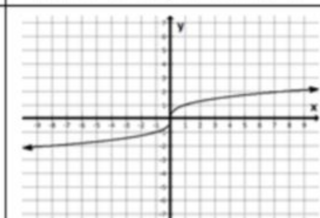
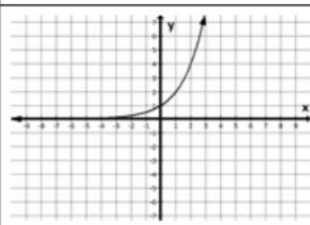
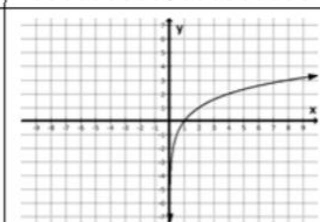
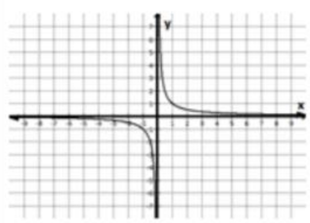
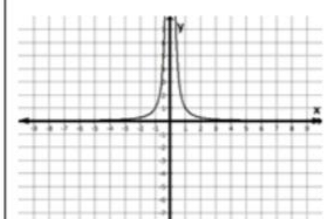
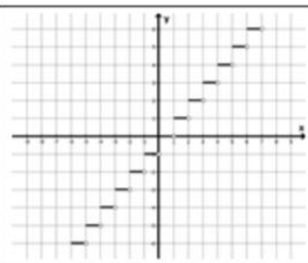
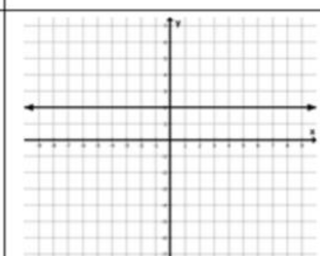
$\csc(-\theta) = -\csc(\theta)$ $\quad \cot(-\theta) = -\cot(\theta)$ $\quad \sec(-\theta) = \sec(\theta)$

Even Functions: Have symmetry about the y-axis. They follow the rule $f(-x) = f(x)$.

Ex. $f(x) = x^2$ $\qquad$ Ex. $f(x) = \frac{1}{x^2+3}$

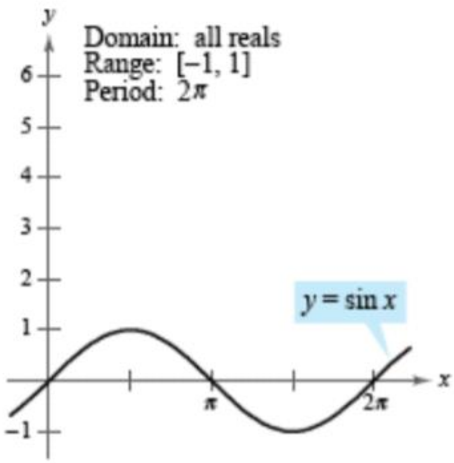
Odd Functions: Have symmetry about the origin. They follow the rule $f(-x) = -f(x)$.

Ex. $f(x) = x^3$ Ex. $f(x) = \sqrt[3]{x}$

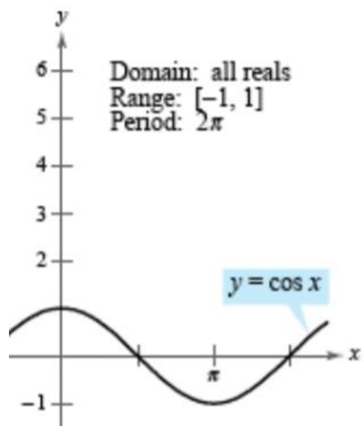
Parent Function	Graph	Parent Function	Graph
$y = x$ Linear Odd Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$		$y = \lvert x \rvert$ Absolute Value Even Domain: $(-\infty, \infty)$ Range: $[0, \infty)$	
$y = x^2$ Quadratic Even Domain: $(-\infty, \infty)$ Range: $[0, \infty)$		$y = \sqrt{x}$ Square Root Neither Domain: $[0, \infty)$ Range: $[0, \infty)$	
$y = x^3$ Cubic Odd Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$		$y = \sqrt[3]{x}$ Cube Root Odd Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$	
$y = b^x,\ b > 1$ Exponential Neither Domain: $(-\infty, \infty)$ Range: $(0, \infty)$		$y = \log_b(x),\ b > 1$ Log Neither Domain: $(0, \infty)$ Range: $(-\infty, \infty)$	
$y = \frac{1}{x}$ Rational or Inverse Odd Domain: $(-\infty, 0) \cup (0, \infty)$ Range: $(-\infty, 0) \cup (0, \infty)$		$y = \frac{1}{x^2}$ Inverse Squared Even Domain: $(-\infty, 0) \cup (0, \infty)$ Range: $(0, \infty)$	
$y = \text{int}(x) = [x]$ Greatest Integer Neither Domain: $(-\infty, \infty)$ Range: $\{y : y \in \mathbb{Z}\}$ (only integers)		$y = C$ Constant Function Even Domain: $(-\infty, \infty)$ Range: $\{y : y = C\}$	

The Greatest Integer graph above is for your reference. You will not have to graph it.

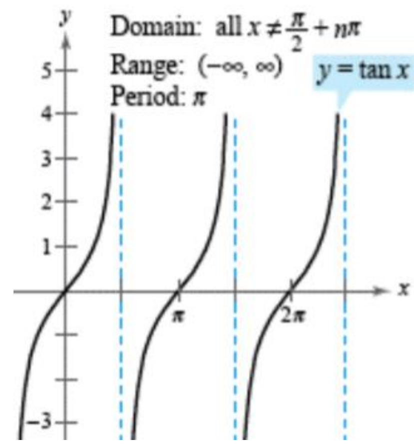
Sine (sin)



Cosine (cos)



Tangent (tan)



Algebra Review

1. Find the equation of the line in point-slope form that is
   a.) parallel to the line $y = 2x - 5$ and passes through the point $(-1,5)$.
   b.) perpendicular to the line $y = 2x - 5$ and passes through the point $(-1,5)$.

For questions 2-4, let $f(x) = \sqrt{x-3}$ and $g(x) = x^3 + 5$. Find:

2. $g(f(x))$

3. $f(g(3))$

4. $g^{-1}(x)$

5. Algebraically find the inverse of $y = \frac{3}{x-2} - 1$.

For questions 6-14, simplify completely.

6. $\frac{\sqrt{x}+x^3}{x}$

7. e^{ln3}

8. $ln1$

9. lne^7

10. $\log_2 32$

11. e^{4lnx}

12. $\frac{4xy^{-2}}{12x^{-\frac{1}{2}}y^{-5}}$

13. $27^{-\frac{2}{3}}$

14. $\frac{3x(x+1)-2(2x+1)}{(x-1)^2}$

15. Rewrite $\frac{1}{2}\ln(x-3)+\ln{(x+2)}$ as a single logarithmic expression.

16. Solve for t. Leave as an exact value.

a. $(1.045)^t = 2$

b. $e^{2t} - 6 = 10$

17. Solve for x. $\log_3 x + \log_3(x-4) = 1$

18. Solve for x. $27^{2x} = 9^{x-3}$

19. Solve for x. $\ln{(3x)} = 16$. Leave as an exact value.

20. Solve for x. $x^3 + 3x^2 - 5x - 15 = 0$

21. Solve for x. $x^4 - 9x^2 + 8 = 0$

For 22-27, find the exact value of the following trig functions.

22. $\sin\left(\frac{7\pi}{6}\right)$

23. $\cos\left(\frac{3\pi}{4}\right)$

24. $\tan\left(\frac{11\pi}{6}\right)$

25. $\cos(\pi)$

26. $\sin\left(\frac{2\pi}{3}\right)$

27. $\tan\left(\frac{5\pi}{4}\right)$

28. Verify: $\frac{sinx}{1-cosx}+\frac{1-cosx}{sinx}=2cscx$

29. Algebraically find the solution of the equation $2\sin^2 x = 1 - sinx$ on $[0,2\pi)$.

30. Algebraically determine all points of intersection. $y = x^2 + 3x - 4$ and $y = 5x + 11$.

31. Use a graphing calculator to estimate the zeros of the function to 3 decimal places. $f(x) = x^3 - 3x^2 + 6x - 1$.

32. Algebraically determine if each function is even, odd, or neither.

a. $y = 2x^2 - 7$

b. $f(x) = -4x^3 - 3x$

c. $f(x) = 2x^4 - x^2 + 6$

33. For the function below, find the x-intercepts, y-intercept, domain, range, VA, HA, and/or holes. Also, provide a rough sketch of the function.

$$f(x) = \frac{x + 3}{2x - 4}$$

34. $\int_1^4 \frac{4x^4-5x^2-8x+1}{2\sqrt{x}}\,dx$

35. $\int \frac{\sin^3(2x)\cos(2x)}{\sqrt{4+\sin^4(2x)}}\,dx$

36. $\int_0^3 (2x-3)^3\,dx$

37. $\int -3e^{2x}\,dx$

38. Find the equation of the tangent line to the graph of $f(x) = x^2 - x + 4$ at the point $(1, -3)$.

Find the derivative of the following:

39. $f(x) = 3\,sin^2(2x), \quad f'(x) =$

40. $y = \frac{1}{2}\tan(\frac{\theta}{3}), \; y'(3) =$

41. $f(x) = -2x^3\,cos(5x), \; f'(x) =$

42. $y = -e^{2x}\,sin\left(\frac{1}{2}x - 1\right), \; y' =$

43. $y = (3x - 1)^3(4x^2 - 3)^2$. Find y' and express answer in simplest factored form.

44. $g(x) = \frac{(8x+3)^2}{(7x-3)^3}$ Find g' and express answer in simplest factored form.

45. $y = \ln(4x)\tan(3x^2)$. Find y'.

46. $2xy^2 - 3x = 5xy$. Find $\frac{dy}{dx}$ (Hint: implicit)

Limits and Continuity.

47. $lim_{x \to 1^+} \frac{|5-5x|}{x-1} =$

48. $lim_{x \to 0} \frac{sinx - cosx}{cos2x} =$

49. $lim_{x \to -\infty} \frac{4x^3 - 6x + 1}{7x^4 + 5} =$

50. $lim_{x \to 0} \frac{\sin(cosx)}{secx}$

51. $lim_{x \to 0^-} \frac{3}{1+e^{\frac{1}{x}}} =$

52. For the following function, find any discontinuities. State whether they are removable or non-removable. If removable, redefine the function so that it will be continuous at that value.

$$f(x) = \begin{cases} 3x - 1; & x > 2 \\ -5; & x = 2 \\ 1 + 2x; & x < 2 \end{cases}$$

53. $lim_{x \to -\infty} \frac{3-x}{\sqrt{2x^2+1} + 5x} =$

54. Find the values of a and b that will make the function $f(x)$ differentiable.

$$f(x) = \begin{cases} ax^2 + 1 & x \geq 1 \\ bx - 3 & x < 1 \end{cases}.$$

55. Find the points on the curve $y = 2x^3 + 3x^2 - 12x + 1$ where the tangent is horizontal.

56. A particle has position function $s(t) = t^3 - 12t^2 + 36t, t \geq 0$, where $t$is measured in seconds and $s(t)$ in meters.

a. Find the velocity at any time t.

b. What is the velocity after 3 seconds?

c. When is the particle at rest?

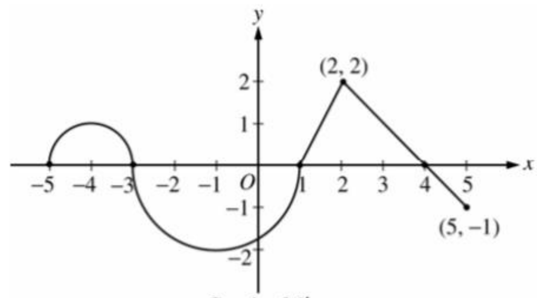
d. Find the acceleration at any time t.

57. The length of a rectangle is decreasing by 2 inches per second and the width is increasing by 3 inches per second. When the length is 10 inches and the width is 6 inches, how fast is the area changing at that instant? (Related Rates)

58. For the function $f(x) = 4x^3 - 12x^2$, find:

a. Intervals of increasing and decreasing
b. Max/Min-state if relative or absolute
c. Intervals of concavity
d. Coordinates of Inflection Points
e. Sketch of the function

59. Let f be a function defined on the closed interval $-5 \le x \le 5$ with $f(1) = 3$. The graph of f', the derivative of f, consists of two semicircles and two line segments, as shown above.



Graph of f'

(a) For $-5 < x < 5$, find all values x at which f has a relative maximum. Justify your answer.

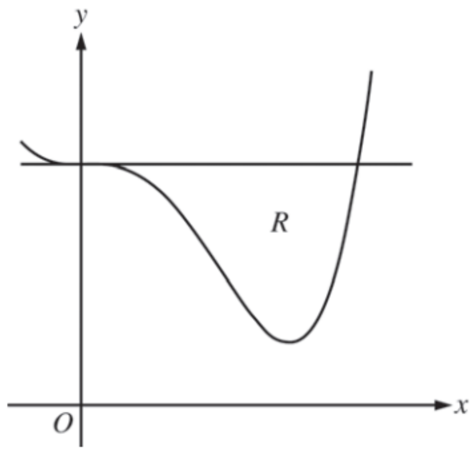
(b) For $-5 < x < 5$, find all values x at which the graph of f has a point of inflection. Justify your answer.

(c) Find all intervals on which the graph of f is concave up and also has positive slope. Explain your reasoning.

(d) Find the absolute minimum value of $f(x)$ over the closed interval $-5 \le x \le 5$. Explain your reasoning.

Calculator Allowed.

60.



Let R be the region enclosed by the graph of $f(x) = x^4 - 2.3x^3 + 4$ and the horizontal line $y = 4$, as shown in the figure above.

(a) Find the volume of the solid generated when R is rotated about the horizontal line $y = -2$.

(b) Region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is an isosceles right triangle with a leg in R. Find the volume of the solid.

(c) The vertical line $x = k$ divides R into two regions with equal areas. Write, but do not solve, an equation involving integral expressions whose solution gives the value k.