## Summer Assignment <br> Calculus

Directions: Show all work for the following problems, even for multiple choice. Do not use a calculator unless you are told to do so. All of this material was learned in Algebra II and in PreCalculus.

Academic Integrity: All work should be completed independently and without the assistance of unapproved resources. Any work violating academic integrity will be subject to a " 0 " and any additional consequences as outlined in the Knox Academic Integrity Policy attached to this assignment.

Due Date: Your work is due the first day of your Calculus class. All late work will be subjected to a grade reduction or penalty as outlined in the course syllabus and copied below:

All major assignments not submitted on the due date will face a $10 \%$ deduction of max points per day for up to five (5) days and up to a $50 \%$ deduction.
Summer Assignments for AP Classes that are not submitted on time will result in the student being dropped from the course.

If you have any questions or concerns regarding this assignment, please contact the Dean of Academics, Mrs. Pergola, at dpergola@knoxschool.org .

## Multiple Choice

1. Susan won $\$ 2,000$ and invested it into an account with an annual interest rate of $3.2 \%$. If her investment were compounded monthly, which expression best represents the value of her investment after $t$ years?
1) $2000(1.003)^{12 t}$
2) $2000(1.032)^{\frac{t}{12}}$
3) $2064^{\frac{t}{12}}$
4) $\frac{2000(1.032)^{t}}{12}$
2. If $\mathrm{f}(x)=\frac{1}{2} x-3$ and $\mathrm{g}(x)=2 x+5$, what is the value of $(g \circ f)(4)$ ?
1) -13
2) 3.5
3) 3
4) 6
3. The accompanying graph is a sketch of the functic $y=\mathrm{f}(x)$ over the interval $0 \leq x \leq 7$.


What is the value of $(f \circ f)(6)$ ?

1) 1
2) 2
3) 0
4) -2
4. What is the solution of the equation $2 \log _{4}(5 x)=3$ ?
1) 6.4
2) 2.56
3) $\frac{9}{5}$
4) $\frac{8}{5}$
5. A solution of the equation $2 x^{2}+3 x+2=0$ is
1) $-\frac{3}{4}+\frac{1}{4} i \sqrt{7}$
2) $-\frac{3}{4}+\frac{1}{4} i$
3) $-\frac{3}{4}+\frac{1}{4} \sqrt{7}$
4) $\frac{1}{2}$
6. A sketch of $r(x)$ is shown below.


An equation for $r(x)$ could be

1) $r(x)=(x-a)(x+b)(x+c)$
2) $r(x)=(x+a)(x-b)(x-c)^{2}$
3) $r(x)=(x+a)(x-b)(x-c)$
4) $r(x)=(x-a)(x+b)(x+c)^{2}$
7. Evan graphed a cubic function, $f(x)=a x^{3}+b x^{2}+c x+d$, and determined the roots of $f(x)$ to be $\pm 1$ and 2 . What is the value of $b$, if $a=1$ ?
1) 1
2) 2
3) -1
4) -2
8. Which equation represents a graph that has a period of $4 \pi$ ?
1) $y=3 \sin \frac{1}{2} x$
2) $y=3 \sin 2 x$
3) $y=3 \sin \frac{1}{4} x$
4) $y=3 \sin 4 x$
9. What are the amplitude and the period of the graph represented by the equation $y=-3 \cos \frac{\theta}{3}$ ?
1) amplitude: -3 ; period: $\frac{\pi}{3}$
2) amplitude: -3 ; period: $6 \pi$
3) amplitude: 3 ; period: $\frac{\pi}{3}$
4) amplitude: 3 ; period: $6 \pi$
10. Identify the type of symmetry (if any) of the graph of the function.

$$
\mathrm{g}(x)=\frac{3 x^{2}}{4 x^{2}+1}
$$

[A] origin symmetry
[B] $x$-axis symmetry
[C] $y$-axis symmetry
[D] no symmetry

## Free Response Questions

11. Multiply and simplify.

$$
\frac{a^{3}-b^{3}}{3 a^{2}+9 a b+6 b^{2}} \cdot \frac{a^{2}+2 a b+b^{2}}{a^{2}-b^{2}}
$$

12. Simplify the following expression. Leave no negative exponents.

$$
\left(\frac{x^{6} y^{-3}}{27 y^{\frac{3}{5}}}\right)^{-\frac{1}{3}}
$$

13. Simply the following radical.
$\sqrt[3]{-54 x^{7} y^{11}}$
14. Solve for $x$. Leave answer as an exact value.
$\left(e^{x}\right)^{4}=e^{5 x-6}$
15. Given: $f(x)=2 x^{2}+x-3$ and $g(x)=x-1$ Express $f(x) \bullet g(x)-[f(x)+g(x)]$ as a polynomial in standard form.
16. Solve for $b$.
$\log b+\log (1+b)=\log (32-3 b)$
17. Solve the equation $4 x^{2}-12 x=7$ algebraically for $x$.
18. Factor completely: $3 a x^{2}-27 a$
19. Factor completely: $5 x^{3}-20 x^{2}-60 x$
20. Verify the identity.
$\frac{\sec \theta \sin \theta}{\tan \theta+\cot \theta}=\sin ^{2} \theta$
21. Graph the following piecewise function.

$$
g(x)=\left\{\begin{array}{r}
5 x, x \leq 3 \\
x^{2}+4, x>3
\end{array}\right.
$$


22. A piecewise function is given below. State the $x$-values of all discontinuities and the type of discontinuity: hole, jump, vertical asymptote. Also state if the discontinuity is removable or non-removable.

23. Graph at least one cycle of the following trig function.
$y=-3 \sin (x-\pi)+1$
24. Solve by rewriting both sides using the same base.
$3^{x-1}=\left(\frac{1}{243}\right)^{x+4}$
25. Algebraically show if the function is even, odd, or neither.
$f(x)=4 x^{3}+5 x-1$
26. Find exact values for $\theta$ on the interval $[0,2 \pi)$ that satisfy the equation:

$$
\cos \theta+2 \sin \theta \cos \theta=0
$$

27. Graph the rational function below without using a calculator. State the equations of any asymptotes, the coordinates or any holes. If the function has a horizontal asymptote, be sure to check to see if it gets crossed.

$$
f(x)=\frac{x+3}{x^{2}+2 x-3}
$$


28. The inverse of the function $f(x)=\frac{x+1}{x-2}$ is
29. Find the exact value of the trig functions below.
a. $\sin \left(\frac{7 \pi}{6}\right)$
b. $\cos \left(\frac{3 \pi}{4}\right)$
c. $\tan \left(\frac{4 \pi}{3}\right)$
d. $\quad \csc \left(\frac{2 \pi}{3}\right)$
30. Graph one period of the cosine function given below.

$$
y=2 \cos \left(\frac{1}{3} x\right)-4
$$

31. Use long division to divide the following polynomials.

$$
\frac{3 x^{3}-4 x^{2}+5 x-5}{x-2}
$$

32. Find the equation of any vertical, horizontal, or slant (oblique) asymptotes.

$$
f(x)=\frac{2 x-1}{x^{2}-3 x-4}
$$

33. Use the graph of the polynomial function below to answer the following questions. Justify your answer.
A. Is the degree of the polynomial even or odd?
B. Is the leading coefficient positive or negative?
C. Is the function even, odd, or neither?
D. Why is $(x+3)^{2}$ a factor of the polynomial?

E. What is the minimum degree of the polynomial?
F. Give one equation that could represent the function in the graph above.
34. Put the equation of the ellipse below into center standard form by completing the square for $x$ and $y$. State the coordinates of the center coordinates of the vertices and foci. Also, graph.

$$
4 x^{2}-40 x+9 y^{2}+36 y=-100
$$

35. Find the Partial Fraction Decomposition of: $\frac{3}{x^{2}-16}$.
