AP Calculus BC Summer Assignment
2021-2022 School Year

Directions: Complete the attached assignment.

Show all work for this assignment. Problems must be done without a calculator unless stated otherwise. This assignment is due on the first day we meet. You will be held accountable for this material during the first week of classes. Ask for help if you need it.

The next two pages contain properties and rules that you should know off the top of your head. I included them for your reference in case you need a refresher.

Properties of Exponents and Logarithms

Exponents

Let a and b be real numbers and m and n be integers. Then the following properties of exponents hold, provided that all of the expressions appearing in a particular equation are defined.

1. $a^m a^n = a^{m+n}$
2. $(a^m)^n = a^{mn}$
3. $(ab)^m = a^m b^m$
4. $\dfrac{a^m}{a^n} = a^{m-n}$, $a \neq 0$
5. $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$, $b \neq 0$
6. $a^{-m} = \dfrac{1}{a^m}$, $a \neq 0$
7. $a^{\frac{1}{n}} = \sqrt[n]{a}$
8. $a^0 = 1$, $a \neq 0$
9. $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

where m and n are integers in properties 7 and 9.

Properties of Logarithms (Recall that logs are only defined for positive values of x.)

For the natural logarithm	For logarithms base a
1. $\ln xy = \ln x + \ln y$	1. $\log_a xy = \log_a x + \log_a y$
2. $\ln \dfrac{x}{y} = \ln x - \ln y$	2. $\log_a \dfrac{x}{y} = \log_a x - \log_a y$
3. $\ln x^y = y \cdot \ln x$	3. $\log_a x^y = y \cdot \log_a x$
4. $\ln e^x = x$	4. $\log_a a^x = x$
5. $e^{\ln x} = x$	5. $a^{\log_a x} = x$

Useful Identities for Logarithms

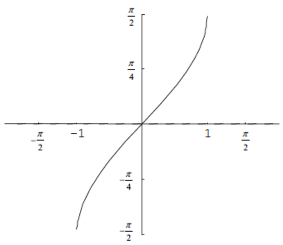
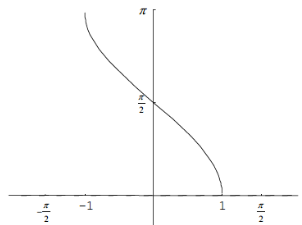
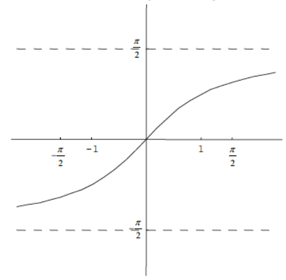
For the natural logarithm	For logarithms base a
1. $\ln e = 1$	1. $\log_a a = 1$, for all $a > 0$
2. $\ln 1 = 0$	2. $\log_a 1 = 0$, for all $a > 0$

If you need a thorough review of logarithms and exponentials, use this link below. The video is great and goes through everything you need to know.

[logs and exponentials]

Please be familiar with these graphs. More so, the domain and range of the graphs.

GRAPHS OF INVERSE TRIG FUNCTIONS

Domain: $[-1,1]$ Range: $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$  $f(x)=\sin^{-1}x$ $f(x)=\arcsin x$	Domain: $[-1,1]$ Range: $[0,\pi]$  $f(x)=\cos^{-1}x$ $f(x)=\arccos x$	Domain: $(-\infty,\infty)$ Range: $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$  $f(x)=\tan^{-1}x$ $f(x)=\arctan x$

In addition to the above material, you also know the 3 Pythagorean identities, the reciprocal trig identities, and all trig values of special angles (unit circle).

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

Ratio Identities

$tan\theta = \frac{sin\theta}{cos\theta}$

$cot\theta = \frac{cos\theta}{sin\theta}$

Reciprocal Identities

$sin\theta = \frac{1}{csc\theta}$ $\quad$ $cos\theta = \frac{1}{sec\theta}$ $\quad$ $tan\theta = \frac{1}{cot\theta}$

$csc\theta = \frac{1}{sin\theta}$ $\quad$ $sec\theta = \frac{1}{cos\theta}$ $\quad$ $cot\theta = \frac{1}{tan\theta}$

Double Angle Identities

$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$

$\cos(2\theta) = 2\cos^2(\theta) - 1$

$\quad\quad = 1 - 2\sin^2(2\theta)$

Even/Odd Identities

$\sin(-\theta) = -\sin(\theta)$ $\quad$ $\tan(-\theta) = -\tan(\theta)$ $\quad$ $\cos(-\theta) = \cos(\theta)$

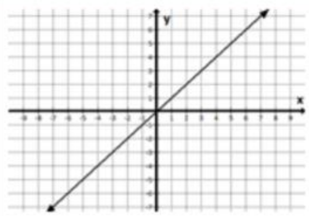
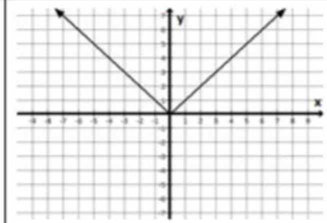
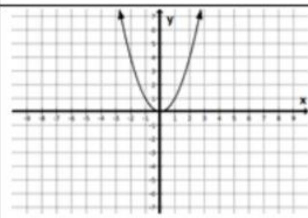
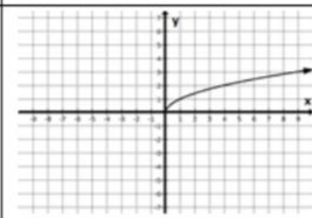
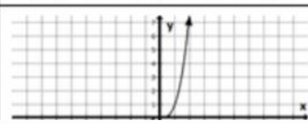
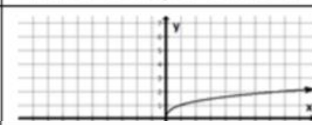
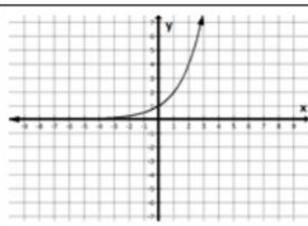
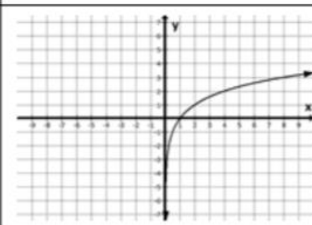
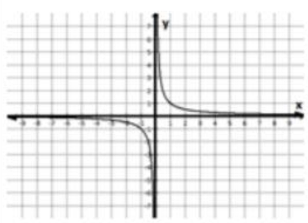
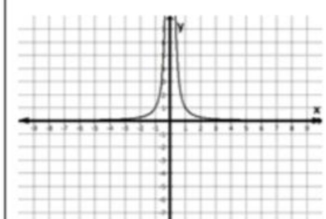
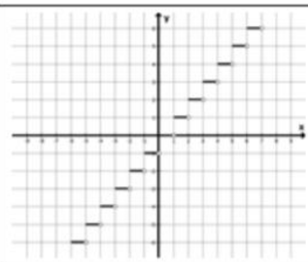
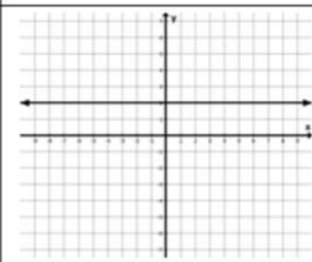
$\csc(-\theta) = -\csc(\theta)$ $\quad$ $\cot(-\theta) = -\cot(\theta)$ $\quad$ $\sec(-\theta) = \sec(\theta)$

Even Functions: Have symmetry about the y-axis. They follow the rule $f(-x) = f(x)$.

Ex. $f(x) = x^2$ $\quad$ Ex. $f(x) = \frac{1}{x^2+3}$

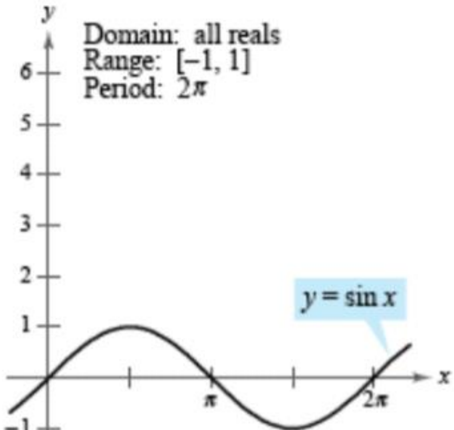
Odd Functions: Have symmetry about the origin. They follow the rule $f(-x) = -f(x)$.

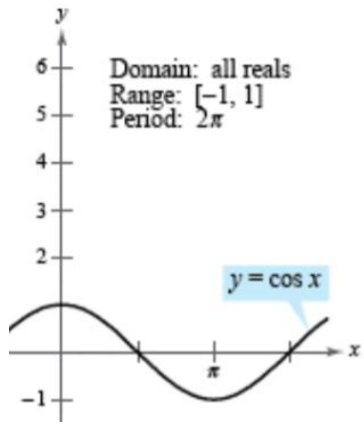
Ex. $f(x) = x^3$ $\quad$ Ex. $f(x) = \sqrt[3]{x}$

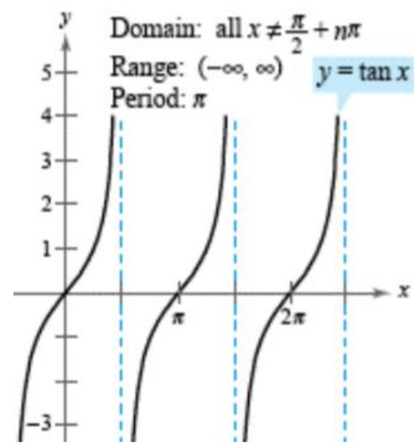
Parent Function	Graph	Parent Function	Graph
$y=x$ Linear Odd Domain: $(-\infty,\infty)$ Range: $(-\infty,\infty)$		$y=\lvert x\rvert$ Absolute Value Even Domain: $(-\infty,\infty)$ Range: $[0,\infty)$	
$y=x^2$ Quadratic Even Domain: $(-\infty,\infty)$ Range: $[0,\infty)$		$y=\sqrt{x}$ Square Root Neither Domain: $[0,\infty)$ Range: $[0,\infty)$	
$y=x^3$ Cubic Odd		$y=\sqrt[3]{x}$ Cube Root Odd	
$y=b^x$, $b>1$ Exponential Neither Domain: $(-\infty,\infty)$ Range: $(0,\infty)$		$y=\log_b(x)$, $b>1$ Log Neither Domain: $(0,\infty)$ Range: $(-\infty,\infty)$	
$y=\frac{1}{x}$ Rational or Inverse Odd Domain: $(-\infty,0)\cup(0,\infty)$ Range: $(-\infty,0)\cup(0,\infty)$		$y=\frac{1}{x^2}$ Inverse Squared Even Domain: $(-\infty,0)\cup(0,\infty)$ Range: $(0,\infty)$	
$y=\text{int}(x)=[x]$ Greatest Integer Neither Domain: $(-\infty,\infty)$ Range: $\{y : y\in\mathbb{Z}\}$ (only integers)		$y=C$ Constant Function Even Domain: $(-\infty,\infty)$ Range: $\{y : y=C\}$	

The Greatest Integer graph above is for your reference. You will not have to graph it.

Sine (sin) **Cosine (cos)** **Tangent (tan)**







Algebra Review

1. Find the equation of the line in point-slope form that is
   a.) parallel to the line $y = 2x - 5$ and passes through the point $(-1,5)$.
   b.) perpendicular to the line $y = 2x - 5$ and passes through the point $(-1,5)$.

For questions 2-4, let $f(x) = \sqrt{x-3}$ and $g(x) = x^3 + 5$. Find:

2. $g(f(x))$

3. $f(g(3))$

4. $g^{-1}(x)$

5. Algebraically find the inverse of $y = \frac{3}{x-2} - 1$.

For questions 6-14, simplify completely.

6. $\frac{\sqrt{x}+x^3}{x}$

7. e^{ln3}

8. $ln1$

9. lne^7

10. $\log_2 32$

11. e^{4lnx}

12. $\frac{4xy^{-2}}{12x^{-\frac{1}{2}}y^{-5}}$

13. $27^{-\frac{2}{3}}$

14. $\frac{3x(x+1)-2(2x+1)}{(x-1)^2}$

15. Rewrite $\frac{1}{2}\ln(x-3)+\ln(x+2)$ as a single logarithmic expression.

16. Solve for t. Leave as an exact value.

a. $(1.045)^t = 2$

b. $e^{2t} - 6 = 10$

17. Solve for $x.$ $\log_3 x + \log_3(x-4) = 1$

18. Solve for x. $27^{2x} = 9^{x-3}$

19. Solve for x. $\ln(3x) = 16$. Leave as an exact value.

20. Solve for x. $x^3 + 3x^2 - 5x - 15 = 0$

21. Solve for x. $x^4 - 9x^2 + 8 = 0$

For 22-27, find the exact value of the following trig functions.

22. $\sin\left(\frac{7\pi}{6}\right)$

23. $\cos\left(\frac{3\pi}{4}\right)$

24. $\tan\left(\frac{11\pi}{6}\right)$

25. $\cos(\pi)$

26. $\sin\left(\frac{2\pi}{3}\right)$

27. $\tan\left(\frac{5\pi}{4}\right)$

28. Verify: $\frac{sinx}{1-cosx} + \frac{1-cosx}{sinx} = 2cscx$

29. Algebraically find the solution of the equation $2\sin^2 x = 1 - sinx$ on $[0,2\pi)$.

30. Algebraically determine all points of intersection. $y = x^2 + 3x - 4$ and $y = 5x + 11$.

31. Use a graphing calculator to estimate the zeros of the function to 3 decimal places. $f(x) = x^3 - 3x^2 + 6x - 1$.

32. Algebraically determine if each function is even, odd, or neither.

a. $y = 2x^2 - 7$

b. $f(x) = -4x^3 - 3x$

c. $f(x) = 2x^4 - x^2 + 6$

33. For the function below, find the x-intercepts, y-intercept, domain, range, VA, HA, and/or holes. Also, provide a rough sketch of the function.

$$f(x) = \frac{x + 3}{2x - 4}$$

Calculus Review

34. $\int_1^4 \frac{4x^4-5x^2-8x+1}{2\sqrt{x}}\,dx$

35. $\int \frac{\sin^3(2x)\cos(2x)}{\sqrt{4+\sin^4(2x)}}\,dx$

36. $\int_0^3 (2x-3)^3\,dx$

37. $\int -3e^{2x}\,dx$

38. Find the equation of the tangent line to the graph of $f(x) = x^2 - x + 4$ at the point $(1, -3)$.

Find the derivative of the following:

39. $f(x) = 3\,sin^2(2x), \quad f'(x) =$

40. $y = \frac{1}{2}\tan(\frac{\theta}{3}),\ y'(3) =$

41. $f(x) = -2x^3\,cos(5x),\ f'(x) =$

42. $y = -e^{2x}\,sin\left(\frac{1}{2}x - 1\right),\ y' =$

43. $y = (3x - 1)^3(4x^2 - 3)^2$. Find y' and express answer in simplest factored form.

44. $g(x) = \frac{(8x+3)^2}{(7x-3)^3}$ Find g' and express answer in simplest factored form.

45. $y = \ln(4x)\tan(3x^2)$. Find y'.

46. $2xy^2 - 3x = 5xy$. Find $\frac{dy}{dx}$ (Hint: implicit)

Limits and Continuity.

47. $lim_{x\to 1^+} \frac{|5-5x|}{x-1} =$

48. $lim_{x\to 0} \frac{sinx - cosx}{cos2x} =$

49. $lim_{x\to-\infty} \frac{4x^3-6x+1}{7x^4+5} =$

50. $lim_{x\to 0} \frac{\sin(cosx)}{secx}$

51. $lim_{x\to 0^-} \frac{3}{1+e^{\frac{1}{x}}} =$

52. For the following function, find any discontinuities. State whether they are removable or non-removable. If removable, redefine the function so that it will be continuous at that value.

$$f(x) = \begin{cases} 3x - 1; & x > 2 \\ -5; & x = 2 \\ 1 + 2x; & x < 2 \end{cases}$$

53. $lim_{x\to-\infty} \frac{3-x}{\sqrt{2x^2+1}+5x} =$

54. Find the values of a and b that will make the function $f(x)$ differentiable.

$$f(x) = \begin{cases} ax^2 + 1 & x \geq 1 \\ bx - 3 & x < 1 \end{cases}.$$

55. Find the points on the curve $y = 2x^3 + 3x^2 - 12x + 1$ where the tangent is horizontal.

56. A particle has position function $s(t) = t^3 - 12t^2 + 36t, t \geq 0$, where $t$is measured in seconds and $s(t)$ in meters.

a. Find the velocity at any time t.

b. What is the velocity after 3 seconds?

c. When is the particle at rest?

d. Find the acceleration at any time t.

e. Find the total distance traveled during the first 8 seconds. You may use your calculator to evaluate the answer.

57. The length of a rectangle is decreasing by 2 inches per second and the width is increasing by 3 inches per second. When the length is 10 inches and the width is 6 inches, how fast is the area changing at that instant? (Related Rates)

58. For the function $f(x) = 4x^3 - 12x^2$, find:

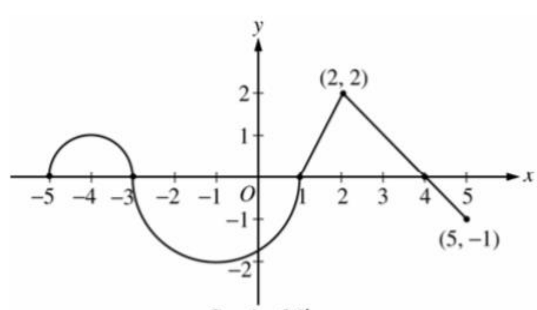
a. Intervals of increasing and decreasing
b. Max/Min-state if relative or absolute
c. Intervals of concavity
d. Coordinates of Inflection Points
e. Sketch of the function

59. Let f be a function defined on the closed interval $-5 \le x \le 5$ with $f(1) = 3$. The graph of f', the derivative of f, consists of two semicircles and two line segments, as shown above.

(a) For $-5 < x < 5$, find all values x at which f has a relative maximum. Justify your answer.

(b) For $-5 < x < 5$, find all values x at which the graph of f has a point of inflection. Justify your answer.

~~(c)~~ Find all intervals on which the graph of f is concave up and also has positive slope. Explain your reasoning.



Graph of f'

(d) Find the absolute minimum value of $f(x)$ over the closed interval $-5 \le x \le 5$. Explain your reasoning.

60. A closed box with a square base must have a volume of 5000 cm^3. Find the dimensions of the bo x that will minimize the amount of material that is used.