AP Calculus AB Summer Assignment
2021-2022 School Year

Directions: Complete the attached assignment.

Show all work for this assignment. Problems must be done without a calculator unless stated otherwise. This assignment is due on the first day we meet. You will be held accountable for this material during the first week of classes. Ask for help if you need it.

The next two pages contain properties and rules that you should know off the top of your head. I included them for your reference in case you need a refresher.

Properties of Exponents and Logarithms

Exponents

Let a and b be real numbers and m and n be integers. Then the following properties of exponents hold, provided that all of the expressions appearing in a particular equation are defined.

1. $a^m a^n = a^{m+n}$
2. $(a^m)^n = a^{mn}$
3. $(ab)^m = a^m b^m$
4. $\dfrac{a^m}{a^n} = a^{m-n}$, $a \neq 0$
5. $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$, $b \neq 0$
6. $a^{-m} = \dfrac{1}{a^m}$, $a \neq 0$
7. $a^{\frac{1}{n}} = \sqrt[n]{a}$
8. $a^0 = 1$, $a \neq 0$
9. $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

where m and n are integers in properties 7 and 9.

Properties of Logarithms (Recall that logs are only defined for positive values of x.)

For the natural logarithm	For logarithms base a
1. $\ln xy = \ln x + \ln y$	1. $\log_a xy = \log_a x + \log_a y$
2. $\ln \dfrac{x}{y} = \ln x - \ln y$	2. $\log_a \dfrac{x}{y} = \log_a x - \log_a y$
3. $\ln x^y = y \cdot \ln x$	3. $\log_a x^y = y \cdot \log_a x$
4. $\ln e^x = x$	4. $\log_a a^x = x$
5. $e^{\ln x} = x$	5. $a^{\log_a x} = x$

Useful Identities for Logarithms

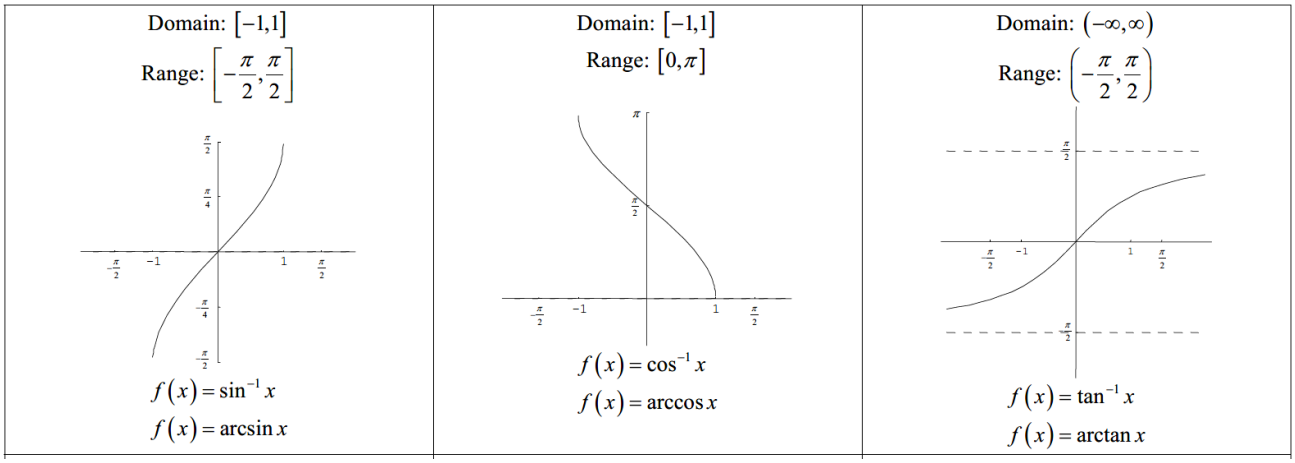
For the natural logarithm	For logarithms base a
1. $\ln e = 1$	1. $\log_a a = 1$, for all $a > 0$
2. $\ln 1 = 0$	2. $\log_a 1 = 0$, for all $a > 0$

If you need a thorough review of logarithms and exponentials, use this link below. The video is great and goes through everything you need to know.

[logs and exponentials]

Please be familiar with these graphs. More so, the domain and range of the graphs.

GRAPHS OF INVERSE TRIG FUNCTIONS



In addition to the above material, you also know the 3 Pythagorean identities, the reciprocal trig identities, and all trig values of special angles (unit circle).

Pythagorean Identities

$\sin^2\theta+\cos^2\theta=1$

$1+\tan^2\theta=\sec^2\theta$

$1+\cot^2\theta=\csc^2\theta$

Ratio Identities

$tan\theta=\frac{sin\theta}{cos\theta}$

$cot\theta=\frac{cos\theta}{sin\theta}$

Reciprocal Identities

$sin\theta=\frac{1}{csc\theta}$ $\quad cos\theta=\frac{1}{sec\theta}$ $\quad tan\theta=\frac{1}{cot\theta}$

$csc\theta=\frac{1}{sin\theta}$ $\quad sec\theta=\frac{1}{cos\theta}$ $\quad cot\theta=\frac{1}{tan\theta}$

Double Angle Identities

$\sin(2\theta)=2\sin(\theta)\cos(\theta)$

$\cos(2\theta)=2\cos^2(\theta)-1$

$=1-2\sin^2(2\theta)$

Even/Odd Identities

$\sin(-\theta)=-\sin(\theta)$ $\quad \tan(-\theta)=-\tan(\theta)$ $\quad \cos(-\theta)=\cos(\theta)$

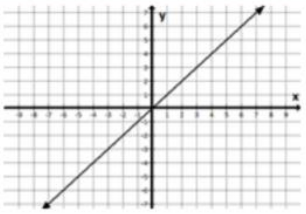
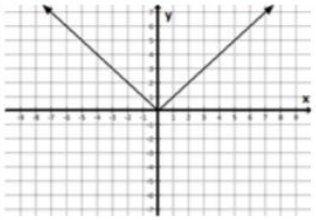
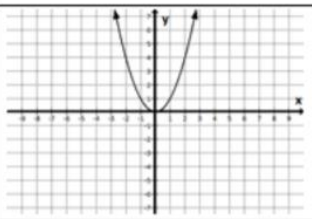
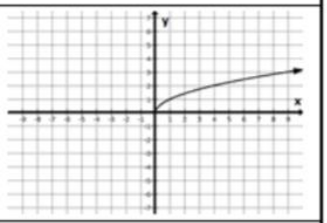
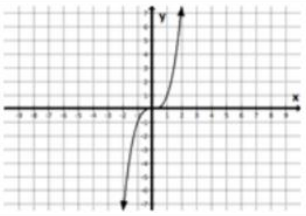
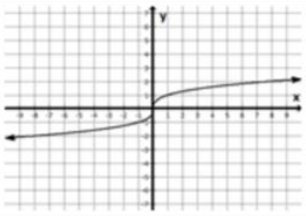
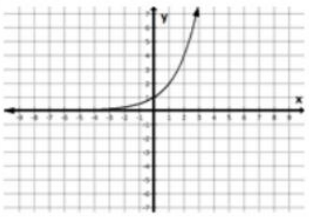
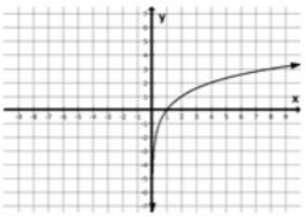
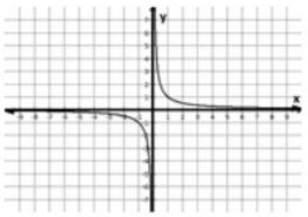
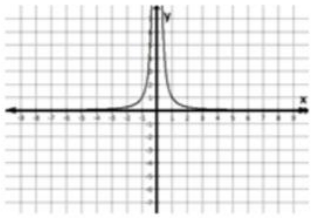
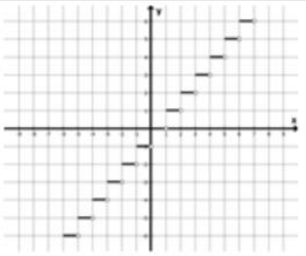
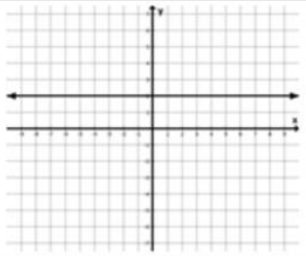
$\csc(-\theta)=-\csc(\theta)$ $\quad \cot(-\theta)=-\cot(\theta)$ $\quad \sec(-\theta)=\sec(\theta)$

Even Functions: Have symmetry about the y-axis. They follow the rule $f(-x) = f(x)$.

Ex. $f(x) = x^2$ Ex. $f(x) = \frac{1}{x^2+3}$

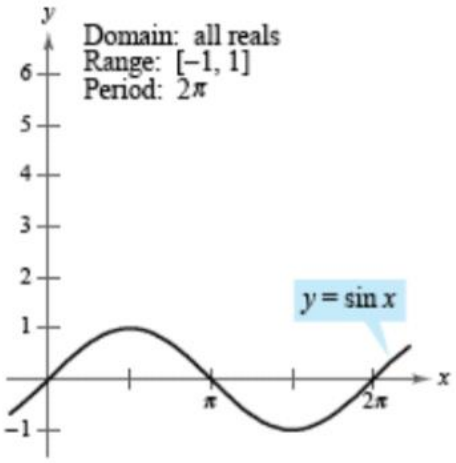
Odd Functions: Have symmetry about the origin. They follow the rule $f(-x) = -f(x)$.

Ex. $f(x) = x^3$ Ex. $f(x) = \sqrt[3]{x}$

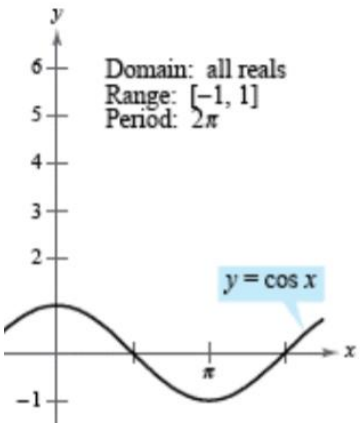
Parent Function	Graph	Parent Function	Graph
$y = x$ Linear Odd Domain: $(-\infty,\infty)$ Range: $(-\infty,\infty)$		$y = \lvert x \rvert$ Absolute Value Even Domain: $(-\infty,\infty)$ Range: $[0,\infty)$	
$y = x^2$ Quadratic Even Domain: $(-\infty,\infty)$ Range: $[0,\infty)$		$y = \sqrt{x}$ Square Root Neither Domain: $[0,\infty)$ Range: $[0,\infty)$	
$y = x^3$ Cubic Odd Domain: $(-\infty,\infty)$ Range: $(-\infty,\infty)$		$y = \sqrt[3]{x}$ Cube Root Odd Domain: $(-\infty,\infty)$ Range: $(-\infty,\infty)$	
$y = b^x,\ b > 1$ Exponential Neither Domain: $(-\infty,\infty)$ Range: $(0,\infty)$		$y = \log_b(x),\ b > 1$ Log Neither Domain: $(0,\infty)$ Range: $(-\infty,\infty)$	
$y = \frac{1}{x}$ Rational or Inverse Odd Domain: $(-\infty,0)\cup(0,\infty)$ Range: $(-\infty,0)\cup(0,\infty)$		$y = \frac{1}{x^2}$ Inverse Squared Even Domain: $(-\infty,0)\cup(0,\infty)$ Range: $(0,\infty)$	
$y = \text{int}(x) = [x]$ Greatest Integer Neither Domain: $(-\infty,\infty)$ Range: $\{y : y \in \mathbb{Z}\}$ (only integers)		$y = C$ Constant Function Even Domain: $(-\infty,\infty)$ Range: $\{y : y = C\}$	

The Greatest Integer graph above is for your reference. You will not have to graph it.

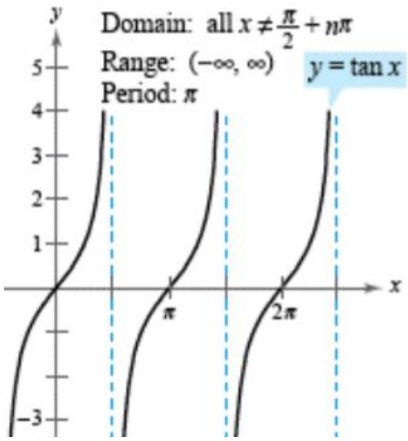
Sine (sin)



Cosine (cos)



Tangent (tan)



Algebra Review

Multiple Choice

1. Susan won $2,000 and invested it into an account with an annual interest rate of 3.2%. If her investment were compounded monthly, which expression best represents the value of her investment after t years?

   1) $2000(1.003)^{12t}$

   2) $2000(1.032)^{\frac{t}{12}}$

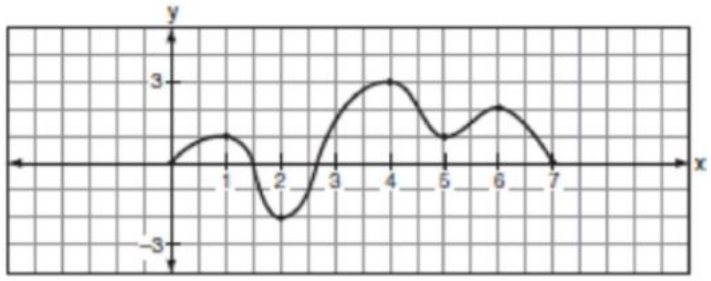
   3) $2064^{\frac{t}{12}}$

   4) $\dfrac{2000(1.032)^{t}}{12}$

2. If $f(x) = \frac{1}{2}x - 3$ and $g(x) = 2x + 5$, what is the value of $(g \circ f)(4)$?

   1) −13
   2) 3.5
   3) 3
   4) 6

3. The accompanying graph is a sketch of the functic $y = f(x)$ over the interval $0 \le x \le 7$.



What is the value of $(f \circ f)(6)$?

1) 1
2) 2
3) 0
4) –2

4. What is the solution of the equation $2\log_4(5x) = 3$?

1) 6.4 2) 2.56 3) $\frac{9}{5}$ 4) $\frac{8}{5}$

5. A solution of the equation $2x^2 + 3x + 2 = 0$ is

1) $-\frac{3}{4} + \frac{1}{4}i\sqrt{7}$

2) $-\frac{3}{4} + \frac{1}{4}i$

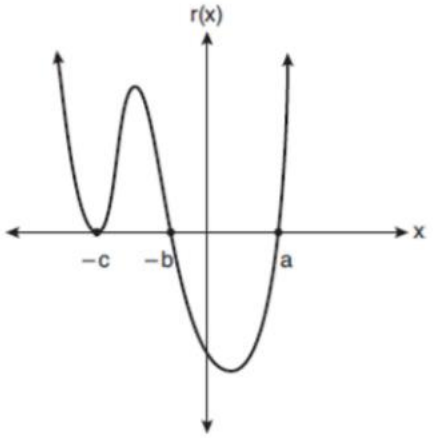
3) $-\frac{3}{4} + \frac{1}{4}\sqrt{7}$

4) $\frac{1}{2}$

6. Evan graphed a cubic function, $f(x) = ax^3 + bx^2 + cx + d$, and determined the roots of $f(x)$ to be ± 1 and 2. What is the value of b, if $a = 1$?

1) 1
2) 2
3) –1
4) –2

7. A sketch of $r(x)$ is shown below.



An equation for $r(x)$ could be

1) $r(x) = (x-a)(x+b)(x+c)$
2) $r(x) = (x+a)(x-b)(x-c)^2$
3) $r(x) = (x+a)(x-b)(x-c)$
4) $r(x) = (x-a)(x+b)(x+c)^2$

8. Which equation represents a graph that has a period of 4π?

1) $y = 3\sin\frac{1}{2}x$ 2) $y = 3\sin 2x$ 3) $y = 3\sin\frac{1}{4}x$ 4) $y = 3\sin 4x$

9. What are the amplitude and the period of the graph represented by the equation $y = -3\cos\frac{\theta}{3}$?

1) amplitude: –3; period: $\frac{\pi}{3}$
2) amplitude: –3; period: 6π
3) amplitude: 3; period: $\frac{\pi}{3}$
4) amplitude: 3; period: 6π

Free Response

10. Find the equation of the line in point-slope form that is
    a.) parallel to the line $y = 2x - 5$ and passes through the point $(-1,5)$.
    b.) perpendicular to the line $y = 2x - 5$ and passes through the point $(-1,5)$.

For questions 11-13, let $f(x) = \sqrt{x-3}$ and $g(x) = x^3 + 5$. Find:

11. $g(f(x))$

12. $f(g(3))$

13. $g^{-1}(x)$

14. Algebraically find the inverse of $y = \frac{3}{x-2} - 1$.

For questions 15-23, simplify completely.

15. $\frac{\sqrt{x}+x^3}{x}$

16. e^{ln3}

17. $ln1$

18. lne^7

19. $\log_2 32$

20. e^{4lnx}

21. $\frac{4xy^{-2}}{12x^{-\frac{1}{2}}y^{-5}}$

22. $27^{-\frac{2}{3}}$

23. $\frac{3x(x+1)-2(2x+1)}{(x-1)^2}$

24. Rewrite $\frac{1}{2}\ln(x-3)+\ln(x+2)$ as a single logarithmic expression.

25. Solve for t. Leave as an exact value.

a. $(1.045)^t = 2$

b. $e^{2t} - 6 = 10$

26. Solve for x. $\log_3 x + \log_3(x-4) = 1$

27. Solve for x. $27^{2x} = 9^{x-3}$

28. Solve for x. $\ln(3x) = 16$. Leave as an exact value.

29. Solve for x. $x^3 + 3x^2 - 5x - 15 = 0$

30. Solve for x. $x^4 - 9x^2 + 8 = 0$

For 31-36 find the exact value of the following trig functions.

31. $\sin\left(\frac{7\pi}{6}\right)$

32. $\cos\left(\frac{3\pi}{4}\right)$

33. $\tan\left(\frac{11\pi}{6}\right)$

34. $\cos(\pi)$

35. $\sin\left(\frac{2\pi}{3}\right)$

36. $\tan\left(\frac{5\pi}{4}\right)$

37. Verify: $\frac{sinx}{1-cosx} + \frac{1-cosx}{sinx} = 2cscx$

38. Algebraically find the solution of the equation $2\sin^2 x = 1 - sinx$ on $[0,2\pi)$.

39. Algebraically determine all points of intersection. $y = x^2 + 3x - 4$ and $y = 5x + 11$.

40. Use a graphing calculator to estimate the zeros of the function to 3 decimal places. $f(x) = x^3 - 3x^2 + 6x - 1$.

41. Use a graphing calculator to find the intersection of $y = 2x^2 - 4x + 1$ and $y = -3x + 4$. Round to three decimal places.

42. Algebraically determine if each function is even, odd, or neither.

a. $y = 2x^2 - 7$

b. $f(x) = -4x^3 - 3x$

c. $f(x) = 2x^4 - x^2 + 6$

43. For the function below, find the x-intercepts, y-intercept, domain, range, VA, HA, and/or holes. Also, provide a rough sketch of the function.

$f(x) = \dfrac{x+3}{2x-4}$

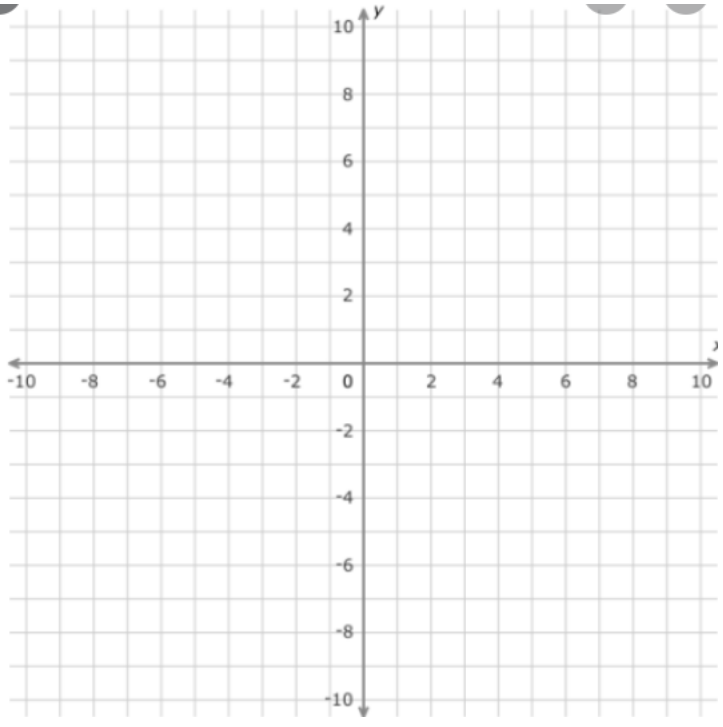
44. Graph $y = (x-2)^2(x+1)(x-3)^3$ by finding the zeros, y-intercept, and end behavior.

45. Simplify the following expression. Leave no negative exponents.

$$\left(\frac{x^6 y^{-3}}{27 y^{\frac{3}{5}}}\right)^{-\frac{1}{3}}$$

46. Graph the piecewise function below.

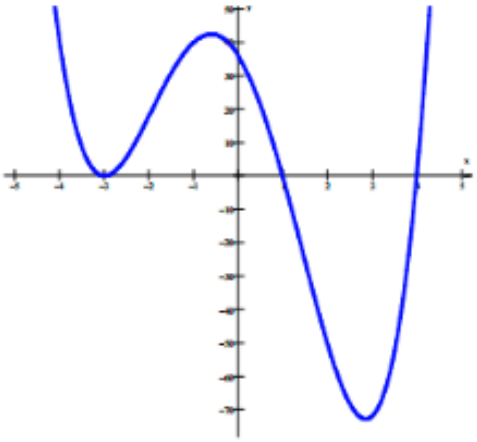
$$g(x) = \begin{cases} 5x, x \le 3 \\ x^2 + 4, x > 3 \end{cases}$$



47. Graph at least one cycle of the following trig function.

$y = -3\sin(x - \pi) + 1$

48. Use the graph of the polynomial function below to answer the following questions. Justify your answer.



A. Is the degree of the polynomial even or odd?

B. Is the leading coefficient positive or negative?

C. Is the function even, odd, or neither?

D. Why is $(x + 3)^2$ a factor of the polynomial?

E. What is the minimum degree of the polynomial?

F. Give one equation that could represent the function in the graph above.

49. Graph the following parabola. Find its vertex, and 4 other points.

$$x = 3(y - 1)^2 - 2$$

50. Put the equation of the circle below into center radius form by completing the square for x and y. State the coordinates of the center and radius.

$$x^2 - 6x - 1 + y^2 + 2y = 5$$