Intermediate Algebra – Winter Assignment

Part 1

During the long winter break, you will be tasked with learning some new and/familiar math concepts. For this assignment, you are being provided various materials to help you to learn and teach yourself some topics, which include Matrices and Radical Expressions. Below is an outline that you should use to help manage your time, as well as stay on track with what needs to be done.

Winter Assignment Outline:

Matrices:

- **12-1: Addition and Subtraction of Matrices**
  - Recall and Review the elimination method for solving systems of equations
  - Introduction to Matrices and how to interpret them
    - Watch this video and follow along with the examples in the video
    - Use page 764 as reference
  - Addition and Subtraction of Matrices
    - Read and follow along with "Problem 1" on page 765
    - Then follow along with the work for "Know-Need-Plan" on page 766
    - Watch this video and follow with the examples
    - Practice Problems: pg 768 #7-10
  - Properties of Adding Matrices
    - Write these down and hold on to them
    - Page 767
      - Closure Property
        - "The addition of two matrices A and B, whose dimensions are $m \times n$, will have a sum of dimensions $m \times n$"
      - Commutative Property
      - Associative Property
      - Additive Identity Property ("Problem 3" on page 766)
      - Additive Inverse Property ("Problem 3" on page 766)
  - Equal matrices and finding unknown matric values – Page 767
    - "Problem 4"
      - Equivalent matrices are when two matrices have the same dimensions, and the corresponding values are equal
    - Practice pg. 768 #28
  - Lesson Check – Page 767
    - Complete all problems 1-6

- 12-2: Multiplication of Matrices
  - Define the word **Scalar**
  - Scalar Multiplication in Matrices – Page 772
    - Scalar Multiplication means to multiply EVERYTHING by a given value
  - Properties of Scalar Multiplication:
    - Write these down
      - Keep in mind all of this is based on the given information:
        "If A and B are $m \times n$, c and d are scalars, and O is the $m \times n$ zero matrix

  - Multiplying Matrices – Page 774
    - Watch this video
    - Write down the Properties of Matrix Multiplication – Page 776

  - Lesson Check – Page 777
    - #1-7
  - Practice – Page 777
    - #12, 13, 27

- Simplifying Radicals
  - Write down the first 12 perfect squares, we did this at the beginning of the year
  - How do you simplify $\sqrt{48}$?
  - Watch video #1
    - This method is prime factorization where you will simplify the number in the radical to represented as the product of prime numbers
  - Watch video #2
    - This is the best way to simplify radicals
  - Watch video #3
    - This is how to simplify radicals with variables and exponents
    - What you need to know:
      - If there is an **even exponent**, it is already a perfect square
        - $\sqrt{x^4} = x^2$
      - If there is an **odd exponent**, take one away to get the perfect square, then add that one back in
        - $\sqrt{x^5} = \sqrt{x^4} \cdot \sqrt{x^1} = x^2\sqrt{x}$

  - Complete the practice problems "Simplifying Radical Expressions" # 1-14

Part 2

Based on the scores for the Unit 3 Exam, you will be tasked with assessing your own work over the break. You will be going over your test and correcting it.

Here is what is expected:

- You have received a blank test on which to perform all of your corrections on
- Look at the answer on the answer key, and perform the necessary work to achieve the correct answer
- Compare your old answer and work to your correct answer and **EXPLAIN** your errors from your original test

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This winter assignment will be counted in your grades in multiple categories. Each of the sections will be counted as a homework grade, and the entire assignment will be averaged together for a test grade. Upon return, there will be review and an assessment on the material covered.

*I will be available on Zoom **EVERY WEDNESDAY** during break. I will be posting alerts as to when I will be online. If my schedule has to change, you will be notified of a new day when I will be available

__Each section will count as a homework grade, and the entire assignment will count as a test grade.__

<u>*There will be no acceptance of late assignments, resulting in a zero</u>**

CHAPTER 12

Matrices

Download videos connecting math to your world.

Interactive! Vary numbers, graphs, and figures to explore math concepts.

The online Solve It will get you in gear for each lesson.

Math definitions in English and Spanish

Online access to stepped-out problems aligned to Common Core

Get and view your assignments online.

Extra practice and review online

Virtual Nerd™ tutorials with built-in support

Chapter Preview

BIG ideas

1 Data Representation

Essential Question How can you use a matrix to organize data?

2 Modeling

Essential Question How can you use a matrix equation to model a real-world situation?

3 Transformations

Essential Question How can a matrix represent a transformation of a geometric figure in the plane?

Vocabulary

English/Spanish Vocabulary Audio Online:

English	Spanish
determinant, *p. 784*	determinante
dilation, *p. 802*	dilatación
equal matrices, *p. 767*	matrices equivalentes
image, *p. 801*	imagen
matrix equation, *p. 765*	ecuación matricial
preimage, *p. 801*	preimagen
scalar multiplication, *p. 772*	multiplicación escalar
variable matrix, *p. 793*	matriz variable
zero matrix, *p. 765*	matriz cero

DOMAINS

- Vector and Matrix Quantities
- Congruence

12-1 Adding and Subtracting Matrices

Common Core State Standards
N-VM.C.8 Add, subtract, and multiply matrices of appropriate dimensions. **Also N-VM.C.10**
MP 1, MP 2, MP 3, MP 4, MP 8

Mathematics Florida Standards
MAFS.912.N-VM.3.8 Add, subtract, and multiply matrices of appropriate dimensions. **Also MAFS.912.N-VM.3.10**
MP 1, MP 2, MP 3, MP 4, MP 8

Objective To add and subtract matrices and to solve matrix equations

If you get stuck, shift your perspective from the patterns in each square to the patterns between squares, or vice versa.

SOLVE IT! Getting Ready!

How can you complete the squares to show number patterns in each square, and from square to square? Explain each pattern.

	2	
4		6
	8	

2		6
	10	
14		18

	6	9
	15	18

16	20	
28	32	

	25	

MATHEMATICAL PRACTICES

In Lesson 3-6, you solved a system of equations by expressing it as a single matrix. Now you will learn how to work with more than one matrix at a time.

Lesson Vocabulary
- corresponding elements
- matrix equation
- zero matrix
- equal matrices

Essential Understanding You can extend the addition and subtraction of numbers to matrices.

Recall that the *dimensions* of a matrix are the numbers of rows and columns. A matrix with 2 rows and 3 columns is a 2×3 matrix. Each number in a matrix is a *matrix element*. In matrix A, a_{12} is the element in row 1 and column 2.

Sometimes you want to combine matrices to get new information. You can combine two matrices with equal dimensions by adding or subtracting the corresponding elements. **Corresponding elements** are elements in the same position in each matrix.

take note — Key Concept: Matrix Addition and Subtraction

To add matrices A and B with the same dimensions, add corresponding elements. Similarly, to subtract matrices A and B with the same dimensions, subtract corresponding elements.

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

$$A + B = \begin{bmatrix} a_{11} + b_{11} & a_{12} + b_{12} \\ a_{21} + b_{21} & a_{22} + b_{22} \end{bmatrix} \qquad A - B = \begin{bmatrix} a_{11} - b_{11} & a_{12} - b_{12} \\ a_{21} - b_{21} & a_{22} - b_{22} \end{bmatrix}$$

Problem 1 Adding and Subtracting Matrices

Think

To add matrices they need to have the same dimensions. What are the dimensions of C?

C has 2 rows and 3 columns, so it's a 2×3 matrix.

Given $C = \begin{bmatrix} 3 & 2 & 4 \\ -1 & 4 & 0 \end{bmatrix}$ and $D = \begin{bmatrix} 1 & 4 & 3 \\ -2 & 2 & 4 \end{bmatrix}$, what are the following?

A $C + D$

$$\begin{bmatrix} 3 & 2 & 4 \\ -1 & 4 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 4 & 3 \\ -2 & 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 3+1 & 2+4 & 4+3 \\ -1+(-2) & 4+2 & 0+4 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 6 & 7 \\ -3 & 6 & 4 \end{bmatrix}$$

B $C - D$

$$\begin{bmatrix} 3 & 2 & 4 \\ -1 & 4 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 4 & 3 \\ -2 & 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 3-1 & 2-4 & 4-3 \\ -1-(-2) & 4-2 & 0-4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -2 & 1 \\ 1 & 2 & -4 \end{bmatrix}$$

Got It? 1. Given $A = \begin{bmatrix} -12 & 24 \\ -3 & 5 \\ -1 & 10 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 1 \\ 2 & -4 \\ -1 & 5 \end{bmatrix}$, what are the following?

a. $A + B$

b. $A - B$

c. Reasoning Is matrix addition commutative? Explain.

A **matrix equation** is an equation in which the variable is a matrix. You can use the addition and subtraction properties of equality to solve a matrix equation. An example of a matrix equation is shown below.

$$\begin{bmatrix} 1 & 0 & 12 \\ 3 & 5 & 9 \\ 7 & 8 & -2 \end{bmatrix} + A = \begin{bmatrix} 8 & 11 & 9 \\ -5 & 5 & 2 \\ 10 & 7 & 8 \end{bmatrix}$$

Problem 2 Solving a Matrix Equation

Sports **The first table shows the teams with the four best records halfway through their season. The second table shows the full season records for the same four teams. Which team had the best record during the second half of the season?**

Records for the First Half of the Season

Team	Wins	Losses
Team 1	30	11
Team 2	29	12
Team 3	25	16
Team 4	24	17

Records for Season

Team	Wins	Losses
Team 1	53	29
Team 2	67	15
Team 3	58	24
Team 4	61	21

Know
- Records for the first half of the season
- Records for the full season

Need
Records for the second half of the season

Plan
- Use the equation: first half records + second half records = season records.
- Solve the matrix equation.

Step 1 Write 4×2 matrices to show the information from the two tables.

Let A = the first half records
B = the second half records
F = the final records

$$A = \begin{bmatrix} 30 & 11 \\ 29 & 12 \\ 25 & 16 \\ 24 & 17 \end{bmatrix} \quad F = \begin{bmatrix} 53 & 29 \\ 67 & 15 \\ 58 & 24 \\ 61 & 21 \end{bmatrix}$$

Think
What are the dimensions of matrix *B*?
B will have 4 rows and 2 columns. It is a 4×2 matrix.

Step 2 Solve $A + B = F$ for B.

$B = F - A$

$$B = \begin{bmatrix} 53 & 29 \\ 67 & 15 \\ 58 & 24 \\ 61 & 21 \end{bmatrix} - \begin{bmatrix} 30 & 11 \\ 29 & 12 \\ 25 & 16 \\ 24 & 17 \end{bmatrix} = \begin{bmatrix} 53-30 & 29-11 \\ 67-29 & 15-12 \\ 58-25 & 24-16 \\ 61-24 & 21-17 \end{bmatrix} = \begin{bmatrix} 23 & 18 \\ 38 & 3 \\ 33 & 8 \\ 37 & 4 \end{bmatrix}$$

Team 2 had the best record (38 wins and 3 losses) during the second half of the season.

Got It? **2.** If $B = \begin{bmatrix} 1 & 6 & -1 \\ 2 & 6 & 1 \\ -1 & -2 & 4 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 0 & 0 \\ -1 & -3 & 6 \\ 2 & 3 & -1 \end{bmatrix}$, and $A - B = C$, what is A?

For $m \times n$ matrices, the additive identity matrix is the **zero matrix** O, or $O_{m \times n}$, with all elements zero. The *opposite*, or *additive inverse*, of an $m \times n$ matrix A is $-A$ where each element is the opposite of the corresponding element of A.

Problem 3 Using Identity and Opposite Matrices

What are the following sums?

Think
How is this like adding real numbers?
Adding zero leaves the matrix unchanged. Adding opposites give you zero.

A $\begin{bmatrix} 1 & 2 \\ 5 & -7 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} 1+0 & 2+0 \\ 5+0 & -7+0 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 5 & -7 \end{bmatrix}$

B $\begin{bmatrix} 2 & 8 \\ -3 & 0 \end{bmatrix} + \begin{bmatrix} -2 & -8 \\ 3 & 0 \end{bmatrix}$

$= \begin{bmatrix} 2+(-2) & 8+(-8) \\ -3+3 & 0+0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Got It? **3.** What are the following sums?

a. $\begin{bmatrix} 14 & 5 \\ 0 & -2 \end{bmatrix} + \begin{bmatrix} -14 & -5 \\ 0 & 2 \end{bmatrix}$

b. $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} -1 & 10 & -5 \\ 0 & 2 & -3 \end{bmatrix}$

take note

Properties — Properties of Matrix Addition

If A, B, and C are $m \times n$ matrices, then

Example	Property
$A + B$ is an $m \times n$ matrix	**Closure Property of Addition**
$A + B = B + A$	**Commutative Property of Addition**
$(A + B) + C = A + (B + C)$	**Associative Property of Addition**
There is a unique $m \times n$ matrix O such that $O + A = A + O = A$	**Additive Identity Property**
For each A, there is a unique opposite, $-A$, such that $A + (-A) = O$	**Additive Inverse Property**

Equal matrices have the same dimensions and equal corresponding elements. For example, $\begin{bmatrix} 0.25 & 1.5 \\ -3 & \frac{4}{5} \end{bmatrix}$ and $\begin{bmatrix} \frac{1}{4} & 1\frac{1}{2} \\ -3 & 0.8 \end{bmatrix}$ are equal matrices. You can use the definition of equal matrices to find unknown values in matrix elements.

Problem 4 Finding Unknown Matrix Values

Multiple Choice **What values of x and y make the equation true?**

$$\begin{bmatrix} 9 & 3x + 1 \\ 2y - 1 & 10 \end{bmatrix} = \begin{bmatrix} 9 & 16 \\ -5 & 10 \end{bmatrix}$$

Ⓐ $x = 3, y = 5$

Ⓑ $x = \frac{17}{3}, y = 5$

Ⓒ $x = 5, y = -2$

Ⓓ $x = 5, y = -3$

Think

How can you solve the equation?
For the two matrices to be equal, the corresponding elements must be equal.

$3x + 1 = 16$	Set corresponding elements equal.	$2y - 1 = -5$
$3x = 16 - 1$	Isolate the variable term.	$2y = -5 + 1$
$3x = 15$	Simplify.	$2y = -4$
$x = 5$	Solve for x and y.	$y = -2$

The correct answer is C.

Got It? **4.** What values of x, y, and z make the following equations true?

a. $\begin{bmatrix} x + 3 & -2 \\ y - 1 & x + 1 \end{bmatrix} = \begin{bmatrix} 9 & -2 \\ 2y + 5 & 7 \end{bmatrix}$

b. $\begin{bmatrix} z & -3 \\ 3x & 0 \end{bmatrix} - \begin{bmatrix} 10 & -4 \\ x & 2y + 6 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 8 & 4y + 12 \end{bmatrix}$

Lesson Check

Do you know HOW?

Find each sum or difference.

1. $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ -4 & 5 \end{bmatrix}$

2. $\begin{bmatrix} 5 & -3 & 7 \\ -1 & 0 & 8 \end{bmatrix} - \begin{bmatrix} 4 & 6 & -1 \\ 2 & 1 & 0 \end{bmatrix}$

Solve each matrix equation.

3. $\begin{bmatrix} 6 & 1 \\ 4 & -2 \end{bmatrix} + X = \begin{bmatrix} 3 & 5 \\ -1 & 9 \end{bmatrix}$

4. $X - \begin{bmatrix} 2 & 0 \\ 5 & -1 \end{bmatrix} = \begin{bmatrix} 4 & 10 \\ 8 & -3 \end{bmatrix}$

Do you UNDERSTAND? MATHEMATICAL PRACTICES

5. Vocabulary Are the two matrices equal? Explain.

$$\begin{bmatrix} \frac{1}{2} & \frac{3}{8} \\ 0.2 & \sqrt[3]{27} \end{bmatrix} \text{ and } \begin{bmatrix} 0.5 & 0.375 \\ \frac{1}{5} & 3 \end{bmatrix}$$

6. Error Analysis Describe and correct the error made in subtracting the two matrices.

$$\begin{bmatrix} 6 \\ 5 \end{bmatrix} - \begin{bmatrix} 3 \\ 7 \end{bmatrix} = \begin{bmatrix} 6 & -3 \\ 5 & -7 \end{bmatrix}$$

Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

Find each sum or difference.

See Problem 1.

7. $\begin{bmatrix} 5 & 4 & 3 \\ 1 & -2 & 6 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

8. $\begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 3 & 2 \\ 3 & 2 & 3 \end{bmatrix}$

9. $\begin{bmatrix} 6.4 & -1.9 \\ -6.4 & 0.8 \end{bmatrix} + \begin{bmatrix} -2.5 & -0.4 \\ 5.8 & 8.3 \end{bmatrix}$

10. $\begin{bmatrix} 1.5 & -1.9 \\ 0 & 4.6 \end{bmatrix} - \begin{bmatrix} 8.3 & -3.2 \\ 2.1 & 5.6 \end{bmatrix}$

Solve each matrix equation.

See Problem 2.

11. $\begin{bmatrix} 1 & 2 \\ 2 & 1 \\ -3 & 4 \end{bmatrix} + X = \begin{bmatrix} 5 & -6 \\ 1 & 0 \\ 8 & 5 \end{bmatrix}$

12. $\begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \end{bmatrix} - X = \begin{bmatrix} 11 & 3 & -13 \\ 15 & -9 & 8 \end{bmatrix}$

13. $X - \begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 5 & -2 \\ 1 & 0 \end{bmatrix}$

14. $X + \begin{bmatrix} 6 & 1 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix}$

Find each sum.

See Problem 3.

15. $\begin{bmatrix} 2 & -3 & 4 \\ 5 & 6 & -7 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

16. $\begin{bmatrix} 6 & -3 \\ -7 & 2 \end{bmatrix} + \begin{bmatrix} -6 & 3 \\ 7 & -2 \end{bmatrix}$

Find the value of each variable.

See Problem 4.

17. $\begin{bmatrix} 2 & 2 \\ -1 & 6 \end{bmatrix} - \begin{bmatrix} 4 & -1 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} x & y \\ -1 & z \end{bmatrix}$

18. $\begin{bmatrix} 2 & 4 \\ 8 & 4.5 \end{bmatrix} = \begin{bmatrix} 4x - 6 & -10t + 5 \\ 4x & 15t + 1.5x \end{bmatrix}$

B Apply

Find each matrix sum or difference if possible. If not possible, explain why.

$$A = \begin{bmatrix} 3 & 4 \\ 6 & -2 \\ 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} -3 & 1 \\ 2 & -4 \\ -1 & 5 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 2 \\ -3 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 5 & 1 \\ 0 & 2 \end{bmatrix}$$

19. $A + B$ **20.** $B + D$ **21.** $C + D$ **22.** $B - A$ **23.** $C - D$

24. Think About a Plan The table shows the number of beach balls produced during one shift at two manufacturing plants. Plant 1 has two shifts per day and Plant 2 has three shifts per day. Write matrices to represent one day's total output at the two plants. Then find the difference between daily production totals at the two plants.

- How can you use the number of shifts to find the total daily production totals at each plant?
- What matrix equation can you use to solve this problem?

Beach Ball Production Per Shift

	1-color		3-color	
	Plastic	Rubber	Plastic	Rubber
Plant 1	500	700	1300	1900
Plant 2	400	1200	600	1600

25. Sports The modern pentathlon is a grueling all-day competition. Each member of a team competes in five events: target shooting, fencing, swimming, horseback riding, and cross-country running. Here are scores for the U.S. women at the 2004 Olympic Games.

a. Write two 5×1 matrices to represent each woman's scores for each event.

b. Find the total score for each athlete.

U.S. Women's Pentathlon Scores, 2004 Olympics

Event	Anita Allen	Mary Beth Iagorashvili
Shooting	952	760
Fencing	720	832
Swimming	1108	1252
Riding	1172	1144
Running	1044	1064

SOURCE: Athens 2004 Olympic Games

26. Data Analysis Refer to the table at the right.

a. Add two matrices to find the total number of people participating in each activity.

b. Subtract two matrices to find the difference between the numbers of males and females in each activity.

c. Reasoning In part (b), does the order of the matrices matter? Explain.

U.S. Participation (millions) in Selected Leisure Activities

Activity	Male	Female
Movies	59.2	65.4
Exercise Programs	54.3	59.0
Sports Events	40.5	31.1
Home Improvement	45.4	41.8

SOURCE: U.S. National Endowment for the Arts

27. Writing Given a matrix A, explain how to find a matrix B such that $A + B = 0$.

Solve each equation for each variable.

28. $\begin{bmatrix} 4b+2 & -3 & 4d \\ -4a & 2 & 3 \\ 2f-1 & -14 & 1 \end{bmatrix} = \begin{bmatrix} 11 & 2c-1 & 0 \\ -8 & 2 & 3 \\ 0 & 3g-2 & 1 \end{bmatrix}$

29. $\begin{bmatrix} 4c & 2-d & 5 \\ -3 & -1 & 2 \\ 0 & -10 & 15 \end{bmatrix} = \begin{bmatrix} 2c+5 & 4d & g \\ -3 & h & f-g \\ 0 & -4c & 15 \end{bmatrix}$

Challenge

30. Find the sum of $E = \begin{bmatrix} 3 \\ 4 \\ 7 \end{bmatrix}$ and the additive inverse of $G = \begin{bmatrix} -2 \\ 0 \\ 5 \end{bmatrix}$.

31. Prove that matrix addition is commutative for 2×2 matrices.

32. Prove that matrix addition is associative for 2×2 matrices.

Standardized Test Prep

SAT/ACT

33. What is the sum $\begin{bmatrix} 5 & 7 & 3 \\ -1 & 0 & -4 \end{bmatrix} + \begin{bmatrix} -7 & 4 & 2 \\ 1 & -2 & -3 \end{bmatrix}$?

(A) The matrices cannot be added.

(B) $\begin{bmatrix} -2 & 11 & 5 \\ 0 & -2 & -7 \end{bmatrix}$ (C) $\begin{bmatrix} 12 & 3 & 1 \\ -2 & 2 & -1 \end{bmatrix}$ (D) $\begin{bmatrix} -35 & 28 & 6 \\ -1 & 0 & 12 \end{bmatrix}$

34. Which arithmetic sequence includes the term 27?

I. $a_1 = 7, a_n = a_{n-1} + 5$ II. $a_n = 3 + 4(n - 1)$ III. $a_n = 57 - 6n$

(F) I only (G) I and II only (H) II and III only (I) I, II, and III

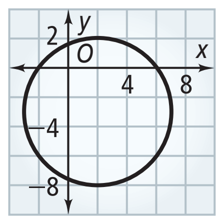
35. Which equation is graphed at the right?

(A) $(x + 3)^2 + (y - 2)^2 = 25$

(B) $(x - 2)^2 + (y + 3)^2 = 25$

(C) $(x + 2)^2 + (y - 3)^2 = 25$

(D) $(x - 3)^2 + (y + 2)^2 = 25$



Short Response

36. For a daily airline flight to Denver, the numbers of checked pieces of luggage are normally distributed with a mean of 380 and a standard deviation of 20. What number of checked pieces of luggage is 3 standard deviations above the mean?

Mixed Review

A set of data with a mean of 62 and a standard deviation of 5 is normally distributed. Find the percent of data within each interval. See Lesson 11-10.

37. from 57 to 67 **38.** greater than 52 **39.** from 62 to 72

Find the slope and y-intercept of each line. See Lesson 2-3.

40. $y = 2x - 6$ **41.** $3y = 6 + 2x$ **42.** $-x - 2y = 12$ **43.** $y = 5x$

Get Ready! **To prepare for Lesson 12-2, do Exercises 44–45.**

Find each sum. See Lesson 12-1.

44. $\begin{bmatrix} 3 & 5 \\ 2 & 8 \end{bmatrix} + \begin{bmatrix} 3 & 5 \\ 2 & 8 \end{bmatrix} + \begin{bmatrix} 3 & 5 \\ 2 & 8 \end{bmatrix}$

45. $\begin{bmatrix} -4 \\ 7 \end{bmatrix} + \begin{bmatrix} -4 \\ 7 \end{bmatrix} + \begin{bmatrix} -4 \\ 7 \end{bmatrix} + \begin{bmatrix} -4 \\ 7 \end{bmatrix} + \begin{bmatrix} -4 \\ 7 \end{bmatrix}$

12-2 Matrix Multiplication

Common Core State Standards

N-VM.C.6 Use matrices to represent and manipulate data . . .

N-VM.C.7 Multiply matrices by scalars to produce new matrices. **Also N-VM.C.8, N-VM.C.9**

MP 1, MP 2, MP 3, MP 4, MP 8

Mathematics Florida Standards

MAFS.912.N-VM.3.6 Use matrices to represent and manipulate data . . .

MAFS.912.N-VM.3.7 Multiply matrices by scalars to produce new matrices. **Also MAFS.912.N-VM.3.8, MAFS.912.N-VM.3.9**

MP 1, MP 2, MP 3, MP 4, MP 8

Objective To multiply matrices using scalar and matrix multiplication

SOLVE IT! **Getting Ready!**

The values in the chart are the recommended 100% daily allowances.

In a family of five, the parents are on a 2000-calorie diet. The three children are on a 2500-calorie diet. For a dessert, the family shares a 500-g cake with 20% fat content. What percentage of the entire family daily fat allowance is in the cake?

	Calories	2,000	2,500
Total Fat	Less than	65 g	80 g
Sat Fat	Less than	20 g	25 g
Cholesterol	Less than	300 mg	300 mg
Sodium	Less than	2,400 mg	2,400 mg
Total Carbohydrate		300 g	375 g
Dietary Fiber		25 g	30 g

MATHEMATICAL PRACTICES

In the Solve It, you may have found the sum of products. Finding the sum of products is essential to matrix multiplication.

Lesson Vocabulary
- scalar
- scalar multiplication

Essential Understanding The product of two matrices is a matrix. To find an element in the product matrix, you multiply the elements of a row from the first matrix by the corresponding elements of a column from the second matrix. Then add the products.

Before you learn how to multiply two matrices, however, you should learn a simpler type of multiplication. This type of multiplication allows you to scale, or resize, the elements of the matrix.

$$3\begin{bmatrix} 5 & -1 \\ 3 & 7 \end{bmatrix} = \begin{bmatrix} 3(5) & 3(-1) \\ 3(3) & 3(7) \end{bmatrix} = \begin{bmatrix} 15 & -3 \\ 9 & 21 \end{bmatrix}$$

The real number factor (such as 3 in the example) is a **scalar**. Multiplication of a matrix A by a scalar c is **scalar multiplication**. To find the resulting matrix cA, you multiply each element of A by c.

take note

Key Concept Scalar Multiplication

To multiply a matrix by a scalar c, multiply each element of the matrix by c.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \qquad cA = \begin{bmatrix} ca_{11} & ca_{12} & ca_{13} \\ ca_{21} & ca_{22} & ca_{23} \end{bmatrix}$$

Problem 1 Using Scalar Products

Think

What operation should you do first?
You should first multiply by the scalars, 4 and 3.

If $A = \begin{bmatrix} 2 & 8 & -3 \\ -1 & 5 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 & 5 \\ 0 & 3 & -2 \end{bmatrix}$, what is $4A + 3B$?

$$4A + 3B = 4\begin{bmatrix} 2 & 8 & -3 \\ -1 & 5 & 2 \end{bmatrix} + 3\begin{bmatrix} -1 & 0 & 5 \\ 0 & 3 & -2 \end{bmatrix}$$
$$= \begin{bmatrix} 8 & 32 & -12 \\ -4 & 20 & 8 \end{bmatrix} + \begin{bmatrix} -3 & 0 & 15 \\ 0 & 9 & -6 \end{bmatrix}$$
$$= \begin{bmatrix} 5 & 32 & 3 \\ -4 & 29 & 2 \end{bmatrix}$$

Got It? **1.** Using matrices A and B from Problem 1, what is $3A - 2B$?

take note

Properties Scalar Multiplication

If A and B are $m \times n$ matrices, c and d are scalars, and O is the $m \times n$ zero matrix, then

Example	Property
cA is an $m \times n$ matrix	Closure Property
$(cd)A = c(dA)$	Associative Property of Multiplication
$c(A + B) = cA + cB$ $(c + d)A = cA + dA$	Distributive Properties
$1 \cdot A = A$	Multiplicative Identity Property
$0 \cdot A = O$ and $cO = O$	Multiplicative Properties of Zero

Problem 2 Solving a Matrix Equation With Scalars

Think

Where have you seen problems that look like this before?
You saw problems like this when you solved one variable equations like $2x + 3(5) = 20$.

What is the solution of $2X + 3\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ 11 & 0 \end{bmatrix}$?

$$2X + \begin{bmatrix} 6 & -3 \\ 9 & 12 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ 11 & 0 \end{bmatrix}$$ Multiply by the scalar 3.

$$2X = \begin{bmatrix} 8 & 5 \\ 11 & 0 \end{bmatrix} - \begin{bmatrix} 6 & -3 \\ 9 & 12 \end{bmatrix}$$ Subtract $\begin{bmatrix} 6 & -3 \\ 9 & 12 \end{bmatrix}$ from each side.

$$2X = \begin{bmatrix} 2 & 8 \\ 2 & -12 \end{bmatrix}$$ Simplify.

$$X = \begin{bmatrix} 1 & 4 \\ 1 & -6 \end{bmatrix}$$ Multiply each side by $\frac{1}{2}$ and simplify.

Got It? **2.** What is the solution of $3X - 2\begin{bmatrix} -1 & 5 \\ 7 & 0 \end{bmatrix} = \begin{bmatrix} 17 & -13 \\ -7 & 0 \end{bmatrix}$?

The product of two matrices is a matrix. To find an element in the product matrix, multiply the elements of a row from the first matrix by the corresponding elements of a column from the second matrix. Then add the products.

take note

Key Concept Matrix Multiplication

To find element c_{ij} of the product matrix AB, multiply each element in the ith row of A by the corresponding element in the jth column of B. Then add the products.

$$AB = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

Problem 3 Multiplying Matrices

Think

What relationship must exist between the numbers of elements in a row of *A* and a column of *B*?

They must be equal.

If $A = \begin{bmatrix} 2 & 1 \\ -3 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 3 \\ 0 & 4 \end{bmatrix}$, what is AB?

Step 1 Multiply the elements in the first row of A by the elements in the first column of B. Add the products and place the sum in the first row, first column of AB.

$$\begin{bmatrix} 2 & 1 \\ -3 & 0 \end{bmatrix}\begin{bmatrix} -1 & 3 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} -2 & \_ \\ \_ & \_ \end{bmatrix} \qquad 2(-1) + 1(0) = -2$$

Step 2 Multiply the elements in the first row of A by the elements in the second column of B. Add the products and place the sum in the first row, second column of AB.

$$\begin{bmatrix} 2 & 1 \\ -3 & 0 \end{bmatrix}\begin{bmatrix} -1 & 3 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} -2 & 10 \\ \_ & \_ \end{bmatrix} \qquad 2(3) + 1(4) = 10$$

Repeat Steps 1 and 2 with the second row of A to fill in row two of the product matrix.

Step 3

$$\begin{bmatrix} 2 & 1 \\ -3 & 0 \end{bmatrix}\begin{bmatrix} -1 & 3 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} -2 & 10 \\ 3 & \_ \end{bmatrix} \qquad (-3)(-1) + 0(0) = 3$$

Step 4

$$\begin{bmatrix} 2 & 1 \\ -3 & 0 \end{bmatrix}\begin{bmatrix} -1 & 3 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} -2 & 10 \\ 3 & -9 \end{bmatrix} \qquad (-3)(3) + 0(4) = -9$$

The product of $\begin{bmatrix} 2 & 1 \\ -3 & 0 \end{bmatrix}$ and $\begin{bmatrix} -1 & 3 \\ 0 & 4 \end{bmatrix}$ is $\begin{bmatrix} -2 & 10 \\ 3 & -9 \end{bmatrix}$.

Got It? **3.** If $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 1 \\ 0 & 2 \end{bmatrix}$, what are the following products?

a. AB

b. BA

c. Reasoning Is matrix multiplication commutative? Explain.

Problem 4 Applying Matrix Multiplication

Sports **In 1966, Washington and New York (Giants) played the highest scoring game in National Football League history. The table summarizes the scoring. A touchdown (TD) is worth 6 points, a field goal (FG) is worth 3 points, a safety (S) is worth 2 points, and a point after touchdown (PAT) is worth 1 point. Using matrix multiplication, what was the final score?**

	TD	FG	S	PAT
WASHINGTON	10	1	0	9
NEW YORK	6	0	0	5

Know
- The number of each type of score
- The point value of each score

Need
The scoring summary and point values as matrices

Plan
Multiply the matrices to find each team's final score.

Think
What is the meaning of each number in matrix *P*?
They are the point values for each type of score.

Step 1 Enter the information in matrices.

$$S = \begin{bmatrix} 10 & 1 & 0 & 9 \\ 6 & 0 & 0 & 5 \end{bmatrix} \qquad P = \begin{bmatrix} 6 \\ 3 \\ 2 \\ 1 \end{bmatrix}$$

Step 2 Use matrix multiplication. The final score is the product SP.

$$SP = \begin{bmatrix} 10 & 1 & 0 & 9 \\ 6 & 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} 6 \\ 3 \\ 2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 10(6) + 1(3) + 0(2) + 9(1) \\ 6(6) + 0(3) + 0(2) + 5(1) \end{bmatrix} = \begin{bmatrix} 72 \\ 41 \end{bmatrix}$$

Step 3 Interpret the product matrix.

The first row of SP shows scoring for Washington, so the final score was Washington 72, New York 41.

Got It? **4.** There are three ways to score in a basketball game: three-point field goals, two-point field goals, and one-point free throws. In 1994, suppose a high school player scored 36 two-point field goals and 28 free throws. In 2006, suppose a high school player scored 7 three-point field goals, 21 two-point field goals, and 18 free throws. Using matrix multiplication, how many points did each player score?

You can multiply two matrices A and B only if the number of columns of A is equal to the number of rows of B.

take note — Property: Dimensions of a Product Matrix

If A is an $m \times n$ matrix and B is an $n \times p$ matrix, then the product matrix AB is an $m \times p$ matrix.

$$\underset{\text{matrix } A}{\text{3 rows}\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}}_{\text{2 columns}} \cdot \underset{\text{matrix } B}{\text{2 rows}\begin{bmatrix} 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 \end{bmatrix}}_{\text{4 columns}}$$

2 columns, 2 rows: equal

3 rows, 4 columns: dimensions of product matrix

Product matrix AB is a 3×4 matrix.

Problem 5 Determining Whether Product Matrices Exist

Think

How can you tell if a product matrix exists without computing it? Compare the dimensions of the matrices.

Does either product AB or BA exist?

$$A = \begin{bmatrix} -2 & 1 \\ 3 & -2 \\ 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} -1 & 0 & 2 & 1 \\ 2 & 0 & 0 & 3 \end{bmatrix}$$

AB

$(3 \times \underbrace{2)(2}_{\text{equal}} \times 4) \rightarrow 3 \times 4$ product matrix

Product AB exists.

BA

$(2 \times \underbrace{4)(3}_{\text{not equal}} \times 2) \rightarrow$ no product

Got It? 5. Do the following products exist?

$$A = \begin{bmatrix} 1 & 4 \\ -3 & 5 \end{bmatrix} \qquad B = \begin{bmatrix} -1 & 1 \end{bmatrix} \qquad C = \begin{bmatrix} 4 & 2 & 0 \\ 1 & 3 & 5 \end{bmatrix}$$

a. AB **b.** BA **c.** AC **d.** CA **e.** BC

Matrix multiplication of square $(n \times n)$ matrices has some of the properties of real number multiplication.

take note — Properties: Matrix Multiplication

If A, B, and C are $n \times n$ matrices, and O is the $n \times n$ zero matrix, then

Example	Property
AB is an $n \times n$ matrix	**Closure Property**
$(AB)C = A(BC)$	**Associative Property of Multiplication**
$A(B + C) = AB + AC$ $(B + C)A = BA + CA$	**Distributive Property**
$OA = AO = O$	**Multiplicative Property of Zero**

Lesson Check

Do you know HOW?

Let $A = \begin{bmatrix} 3 & -1 \\ 2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ -2 & 2 \end{bmatrix}$.

Find each of the following.

1. $2A$

2. $3B - 2A$

3. AB

4. BA

Do you UNDERSTAND? MATHEMATICAL PRACTICES

5. Vocabulary Which type of multiplication, *scalar* or *matrix*, can help you with a repeated matrix addition problem? Explain.

6. Error Analysis Your friend says there is a right order and a wrong order when multiplying A (a 2×4 matrix) and B (a 3×6 matrix). Explain your friend's error.

Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

Use matrices A, B, C, and D. Find each product, sum, or difference. See Problem 1.

$$A = \begin{bmatrix} 3 & 4 \\ 6 & -2 \\ 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} -3 & 1 \\ 2 & -4 \\ -1 & 5 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 2 \\ -3 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 5 & 1 \\ 0 & 2 \end{bmatrix}$$

7. $3A$

8. $4B$

9. $-3C$

10. $-D$

11. $A - 2B$

12. $3A + 2B$

13. $4C + 3D$

14. $2A - 5B$

Solve each matrix equation. Check your answers. See Problem 2.

15. $3\begin{bmatrix} 2 & 0 \\ -1 & 5 \end{bmatrix} - 2X = \begin{bmatrix} -10 & 5 \\ 0 & 17 \end{bmatrix}$

16. $4X + \begin{bmatrix} 1 & 3 \\ -7 & 9 \end{bmatrix} = \begin{bmatrix} -3 & 11 \\ 5 & -7 \end{bmatrix}$

17. $\frac{1}{2}X + \begin{bmatrix} 4 & -3 \\ 12 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

18. $5X - \begin{bmatrix} 1.5 & -3.6 \\ -0.3 & 2.8 \end{bmatrix} = \begin{bmatrix} 0.2 & 1.3 \\ -5.6 & 1.7 \end{bmatrix}$

Find each product. See Problem 3.

19. $\begin{bmatrix} -3 & 4 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 2 & -3 \end{bmatrix}$

20. $\begin{bmatrix} 1 & 0 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} -3 & 4 \\ 5 & 2 \end{bmatrix}$

21. $\begin{bmatrix} 0 & 2 \\ -4 & 0 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ -4 & 0 \end{bmatrix}$

22. $\begin{bmatrix} -3 & 5 \end{bmatrix}\begin{bmatrix} -3 \\ 5 \end{bmatrix}$

23. $\begin{bmatrix} -3 & 5 \end{bmatrix}\begin{bmatrix} -3 & 0 \\ 5 & 0 \end{bmatrix}$

24. $\begin{bmatrix} -3 & 5 \end{bmatrix}\begin{bmatrix} 0 & -3 \\ 0 & 5 \end{bmatrix}$

25. $\begin{bmatrix} 0 & -3 \\ 0 & 5 \end{bmatrix}\begin{bmatrix} -3 & 0 \\ 5 & 0 \end{bmatrix}$

26. $\begin{bmatrix} 1 & 0 \\ -1 & -5 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

27. $\begin{bmatrix} -1 & 3 & -3 \\ 2 & -2 & 1 \end{bmatrix}\begin{bmatrix} 5 \\ 4 \\ 3 \end{bmatrix}$

28. Business A florist makes three special floral arrangements. One uses three lilies. The second uses three lilies and four carnations. The third uses four daisies and three carnations. Lilies cost \$2.15 each, carnations cost \$.90 each, and daisies cost \$1.30 each. See Problem 4.

a. Write a matrix to show the number of each type of flower in each arrangement.

b. Write a matrix to show the cost of each type of flower.

c. Find the matrix showing the cost of each floral arrangement.

Simplifying Radical Expressions

Name____

Date________________

Simplify.

1) $\sqrt{125n}$

2) $\sqrt{216v}$

3) $\sqrt{512k^2}$

4) $\sqrt{512m^3}$

5) $\sqrt{216k^4}$

6) $\sqrt{100v^3}$

7) $\sqrt{80p^3}$

8) $\sqrt{45p^2}$

9) $\sqrt{147m^3n^3}$

10) $\sqrt{200m^4n}$

11) $\sqrt{75x^2y}$

12) $\sqrt{64m^3n^3}$

13) $\sqrt{16u^4v^3}$

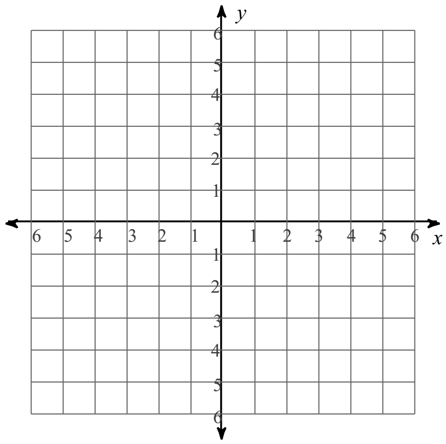
14) $\sqrt{28x^3y^3}$

Name ________________________________ Unit 3 Test

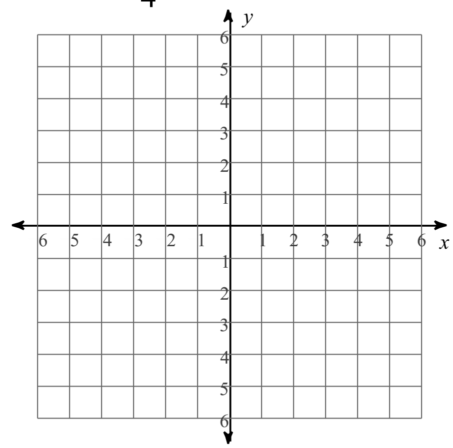
Date________________

Sketch the graph of each line.

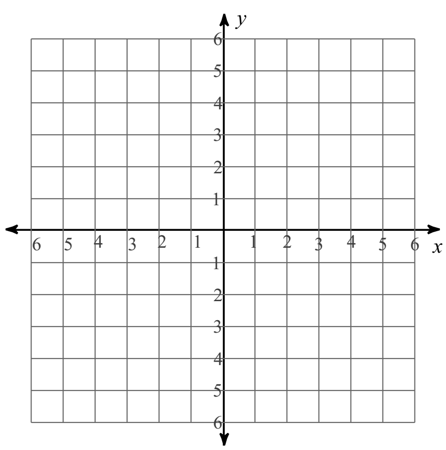
1) $x - 4 = -y$



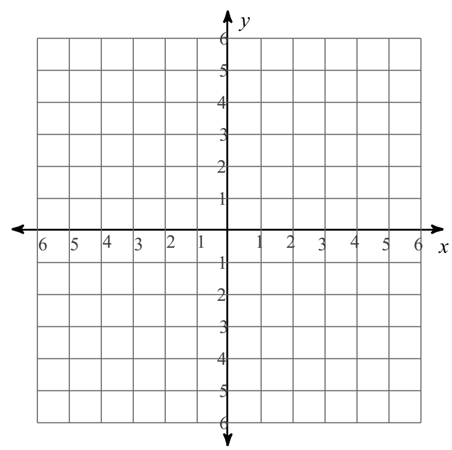
2) $2 + y + \frac{1}{4}x = 0$



3) $5y - 25 = -3x$



4) $-3y + 12 + 4x = 0$



Write the slope-intercept form of the equation of the line through the given point with the given slope.

5) through: $(-2, 0)$, slope $= -2$

6) through: $(-2, -5)$, slope $= 0$

7) through: $(1, 5)$, slope $= 3$

Write the slope-intercept form of the equation of the line through the given points.

8) through: $(0, 2)$ and $(-4, 5)$

9) through: $(-5, -3)$ and $(0, -5)$

Write the point-slope form of the equation of the line through the given points.

10) through: $(1, 4)$ and $(-3, 2)$

11) through: $(-5, -5)$ and $(-2, 5)$

Sovle each system of equations using either substitution method or elimination method.

12) $5x - y = -12$
$-6x + 2y = 16$

13) $-7x + 7y = 35$
$-10x - 4y = -48$

14) $-2x + 6y = 16$
$-4x - 8y = 32$

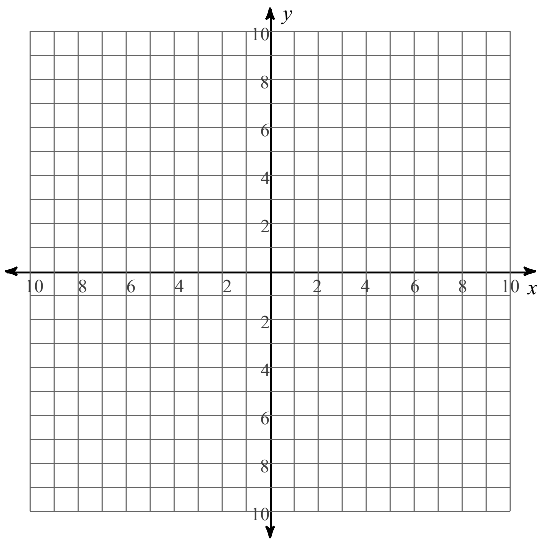
15) $3x - 3y + 2z = -12$
$-x - 6y + z = -14$
$-3x - 2y = 4$

16) $-3x - 5y = 24$
$-x + 3y - 4z = -22$
$5x + 2y + z = -17$

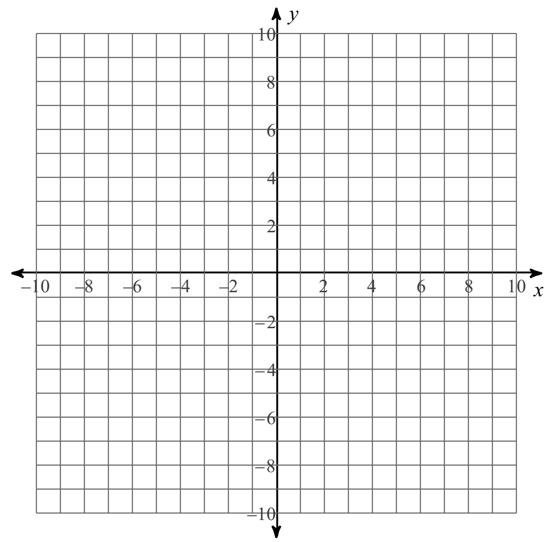
17) $3y + 4z = -15$
$6x + 5y + z = 5$
$6x + 4y + 5z = 10$

Convert the given equations to slope-intercept form, then solve each system by graphing.

18) $8x = 7y - 28$
$2x + 42 = -7y$

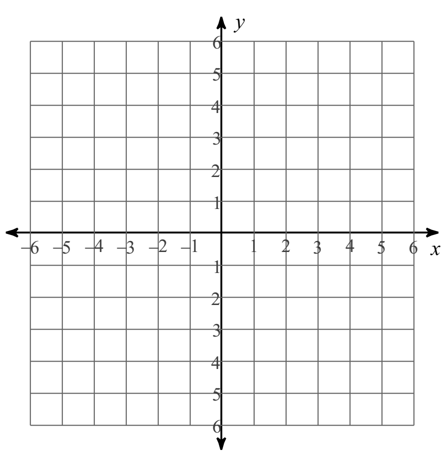


19) $-2x - y = 9$
$-3 - 10x = -y$

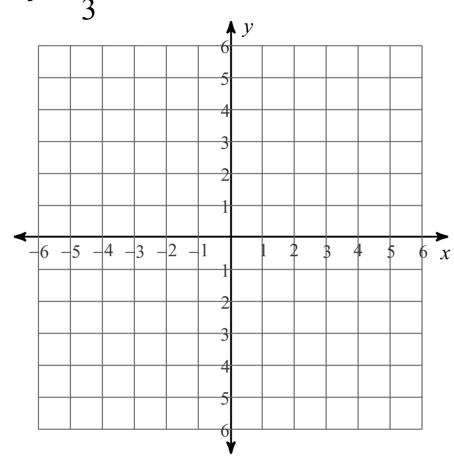


Sketch the graph of each linear inequality.
Convert the given inequality to slope-intercept form.

20) $4x + y \leq 0$

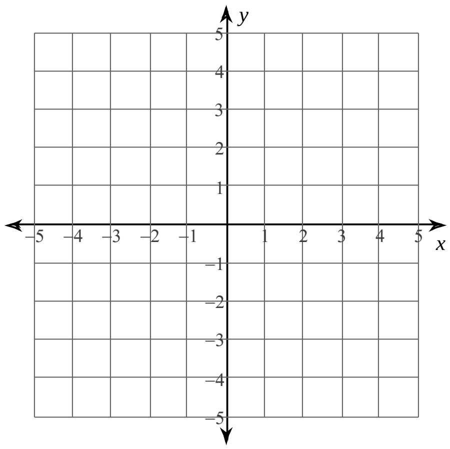


21) $y > \frac{4}{3}x - 4$

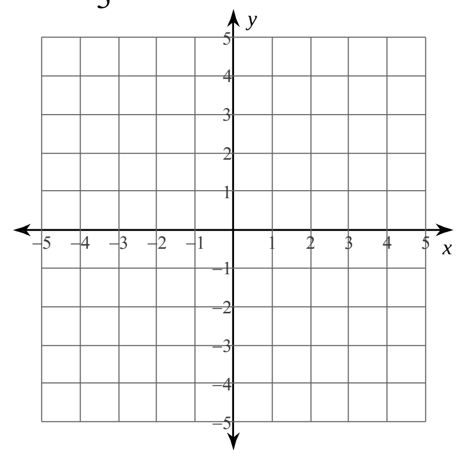


Sketch the solution to each system of inequalities.

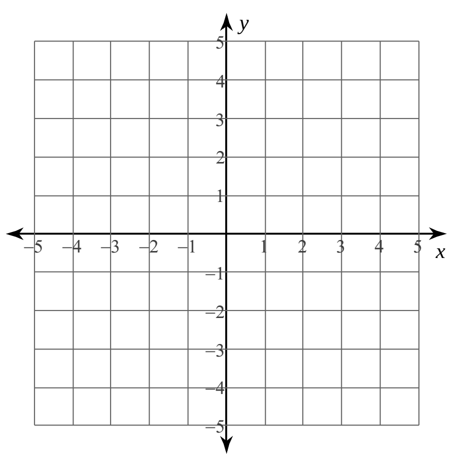
22) $y > -x - 2$
$y < -5x + 2$



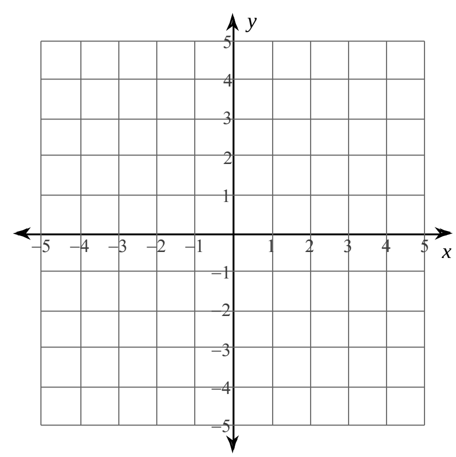
23) $x \leq -3$
$y < \frac{5}{3}x + 2$



24) $3x + 2y \geq -2$
$x + 2y \leq 2$



25) $y \geq \frac{2}{3}x + 3$
$y > -\frac{4}{3}x - 3$



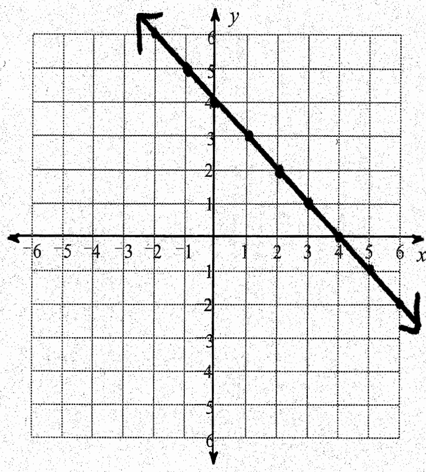
Name Answer key

Unit 3 Test

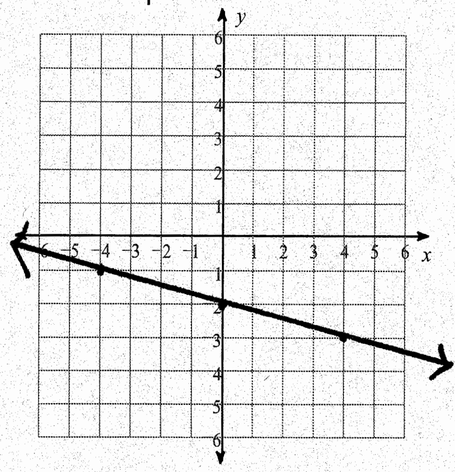
Date____________

Sketch the graph of each line.

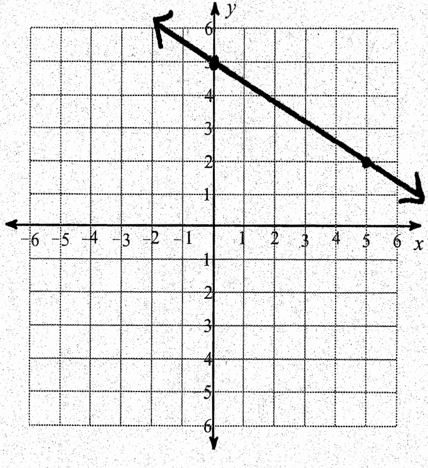
1) $x - 4 = -y$



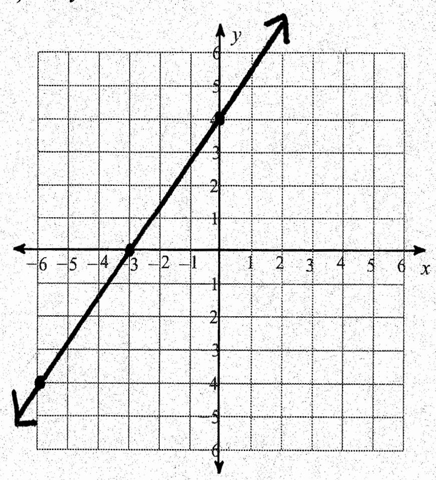
2) $2 + y + \frac{1}{4}x = 0$



3) $5y - 25 = -3x$



4) $-3y + 12 + 4x = 0$



Write the slope-intercept form of the equation of the line through the given point with the given slope.

5) through: $(-2, 0)$, slope $= -2$

$y = -2x - 4$

6) through: $(-2, -5)$, slope $= 0$

$y = 5$

7) through: $(1, 5)$, slope $= 3$

$y = 3x + 2$

Write the slope-intercept form of the equation of the line through the given points.

8) through: $(0, 2)$ and $(-4, 5)$

$y = -\frac{3}{4}x + 2$

9) through: $(-5, -3)$ and $(0, -5)$

$y = \frac{2}{5}x - 5$

Write the point-slope form of the equation of the line through the given points.

10) through: $(1, 4)$ and $(-3, 2)$

$y - 4 = \frac{1}{2}(x - 1)$

11) through: $(-5, -5)$ and $(-2, 5)$

$y + 5 = \frac{10}{3}(x + 5)$

Sovle each system of equations using either substitution method or elimination method.

12) $5x - y = -12$
$-6x + 2y = 16$

$x = -2$
$y = 2$

13) $-7x + 7y = 35$
$-10x - 4y = -48$

$x = 2$
$y = 7$

14) $-2x + 6y = 16$
$-4x - 8y = 32$

$x = 0$
$y = -8$

15) $3x - 3y + 2z = -12$
$-x - 6y + z = -14$
$-3x - 2y = 4$

$x = -4$
$y = 4$
$z = 6$

16) $-3x - 5y = 24$
$-x + 3y - 4z = -22$
$5x + 2y + z = -17$

$x = -3$
$y = -3$
$z = 4$

17) $3y + 4z = -15$
$6x + 5y + z = 5$
$6x + 4y + 5z = 10$

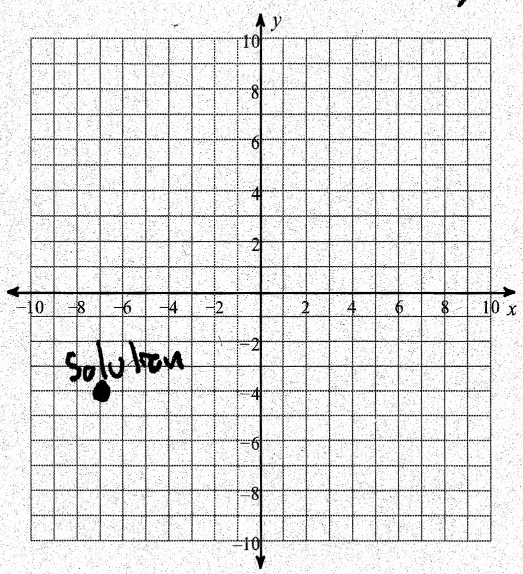
$x = 5$
$y = -5$
$z = 0$

I only gave the solutions, you MUST graph the lines

Convert the given equations to slope-intercept form, then solve each system by graphing.

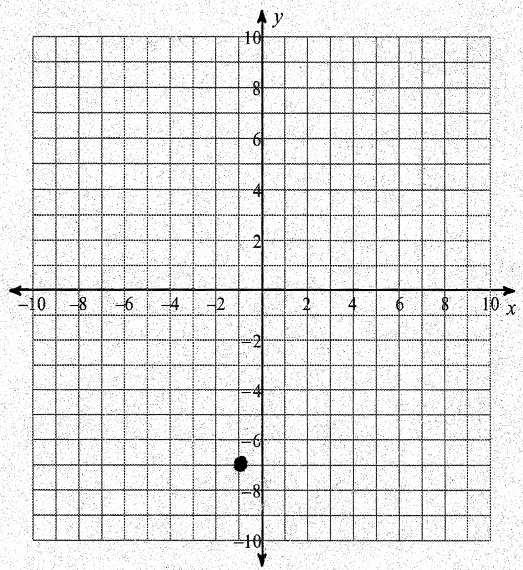
18) $8x = 7y - 28$
$2x + 42 = -7y$

Solution
$(-7, -4)$



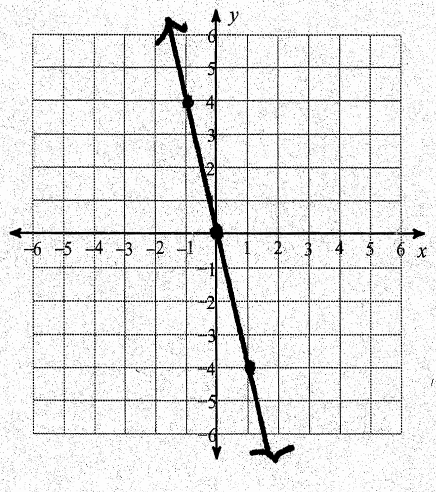
19) $-2x - y = 9$
$-3 - 10x = -y$

Solution
$(-1, -7)$

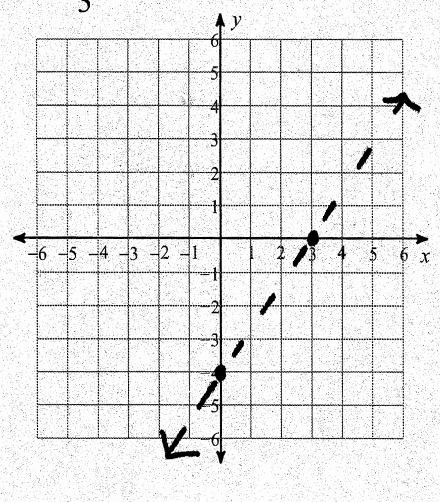


Sketch the graph of each linear inequality.
Convert the given inequality to slope-intercept form.

20) $4x + y \leq 0$



21) $y > \frac{4}{3}x - 4$

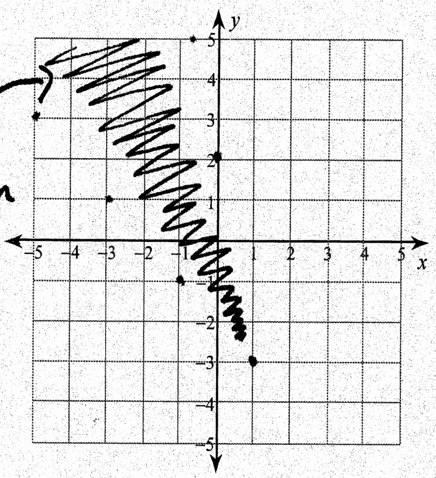


I only shaded the solutions, you **MUST** graph the lines

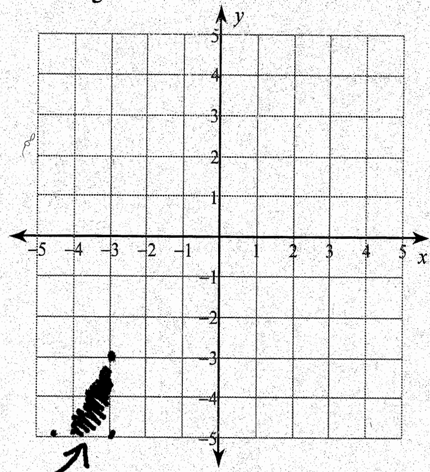
Sketch the solution to each system of inequalities.

22) $y > -x - 2$
$y < -5x + 2$

Solution

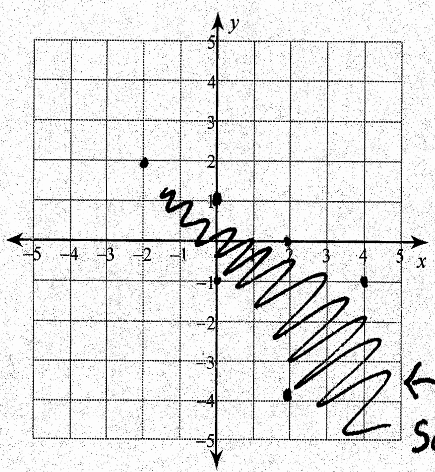


23) $x \le -3$
$y < \frac{5}{3}x + 2$



Solution

24) $3x + 2y \ge -2$
$x + 2y \le 2$



Solution

25) $y \ge \frac{2}{3}x + 3$
$y > -\frac{4}{3}x - 3$

Solution