Geometry – Winter Assignment

During the winter break, you are tasked with exploring and learning **Unit 4: Congruent Triangles** on your own. In this assignment, I have everything broken down by unit, exercises, and videos to help you learn the material at your own pace. The outline below should act as a guide for you to manage your time efficiently. As long as you put in the effort, you can finish this assignment in a good amount of time, allowing you to enjoy the rest of your break.

Winter Assignment Outline:

- 4-1: Congruent Triangles (Estimated time: 30-50 Minutes)
  - Think About a Plan
  - Practice Problems: pg. 221 #1-7
- 4-2: Triangle Congruence by SSS and SAS (Estimated time: 25-40 Minutes)
  - Think About a Plan
  - Practice Problems: pg. 230 #1-7
- 4-3: Triangle Congruence by ASA and AAS (Estimated time: 30-50 Minutes)
  - Think About a Plan
  - Practice Problems: pg. 238 #1-7
- 4-4: Using Corresponding Parts of Congruent Triangles (Estimated time: 15-30 Minutes)
  - Think About a Plan
  - Practice Problems: pg. 246 #1-4
- 4-5: Isosceles and Equilateral Triangles (Estimated time: 30-45 Minutes)
  - Think About a Plan
  - Practice Problems: pg. 253 #1-5
- 4-6: Congruence in Right Triangles (Estimated time: 40-55 Minutes)
  - Think About a Plan
  - Practice Problems: pg. 261 #1-7
- 4-7: Congruence in Overlapping Triangles (Estimated time: 30-45 Minutes)
  - Think About a Plan
  - Practice Problems: pg. 268 #1-7

Here is a YouTube playlist of all of the videos for this assignment: Geometry Unit 4: Congruent Triangles

Estimated minimum time to complete: 3 hours 20 minutes

Estimated maximum time to complete: 5 hours 15 minutes

This assignment will be counted in your grades in multiple categories. Each of the 7 sections will be counted as a homework grade, and the entire assignment will be averaged together for a test grade. Upon return, there will be review and an assessment on the material covered.

*I will be available on Zoom **EVERY WEDNESDAY** during break. I will be posting alerts as to when I will be online. If my schedule has to change, you will be notified of a new day when I will be available

**Each section will count as a homework grade, and the entire assignment will count as a test grade.

***There will be no acceptance of late assignments, resulting in a zero

Practice Problems: pg. 221 #1-7	
1)	2)
3)	4)
5)	6)
7)	

Practice Problems: pg. 230 #1-7	
1)	2)
3)	4)
5)	6)
7)	

Practice Problems: pg. 238 #1-7	
1)	2)
3)	4)
5)	6)
7)	

Practice Problems: pg. 246 #1-4	
1)	2)
3)	4)

Practice Problems: pg. 253 #1-5	
1)	2)
3)	4)
5)	

Practice Problems: pg. 261 #1-7	
1)	2)
3)	4)
5)	6)
7)	

Practice Problems: pg. 268 #1-7	
1)	2)
3)	4)
5)	6)
7)	

CHAPTER 4

Congruent Triangles

Download videos connecting math to your world.

Interactive! Vary numbers, graphs, and figures to explore math concepts.

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Math definitions in English and Spanish

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Chapter Preview

4-1 Congruent Figures
4-2 Triangle Congruence by SSS and SAS
4-3 Triangle Congruence by ASA and AAS
4-4 Using Corresponding Parts of Congruent Triangles
4-5 Isosceles and Equilateral Triangles
4-6 Congruence in Right Triangles
4-7 Congruence in Overlapping Triangles

BIG ideas

1 Visualization

Essential Question How do you identify corresponding parts of congruent triangles?

2 Reasoning and Proof

Essential Question How do you show that two triangles are congruent?

3 Reasoning and Proof

Essential Question How can you tell whether a triangle is isosceles or equilateral?

Vocabulary

English/Spanish Vocabulary Audio Online:

English	Spanish
base angles of an isosceles triangle, *p. 250*	ángulos de base de un triángulo isósceles
base of an isosceles triangle, *p. 250*	base de un triángulo isósceles
congruent polygons, *p. 219*	polígonos congruentes
corollary, *p. 252*	corolario
hypotenuse, *p. 258*	hipotenusa
legs of an isosceles triangle, *p. 250*	catetos de un triángulo isósceles
legs of a right triangle, *p. 258*	catetos de un triángulo rectángulo
vertex angle of an isosceles triangle, *p. 250*	ángulo en vértice de un triángulo isósceles

DOMAINS

- Congruence
- Mathematical Practice: Construct viable arguments
- Modeling with Geometry

4-1 Congruent Figures

Mathematics Florida Standards

Prepares for MAFS.912.G-SRT.2.5 Use congruence . . . criteria for triangles to solve problems and to prove relationships in geometric figures.

MP 1, MP 3, MP 4, MP 7

Objective To recognize congruent figures and their corresponding parts

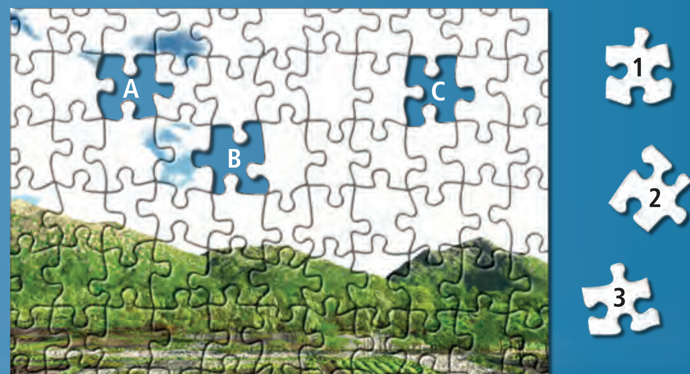
Having trouble? How can tracing pieces 1, 2, and 3 help?

MATHEMATICAL PRACTICES

SOLVE IT!

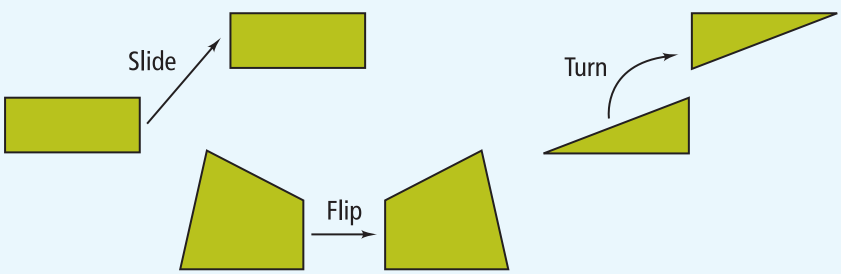
Getting Ready!

You are working on a puzzle. You've almost finished, except for a few pieces of the sky. Place the remaining pieces in the puzzle. How did you figure out where to place the pieces?



Lesson Vocabulary
- congruent polygons

Congruent figures have the same size and shape. When two figures are congruent, you can slide, flip, or turn one so that it fits exactly on the other one, as shown below. In this lesson, you will learn how to determine if geometric figures are congruent.



Essential Understanding You can determine whether two figures are congruent by comparing their corresponding parts.

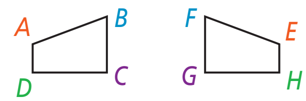
take note

Key Concept Congruent Figures

Definition

Congruent polygons have congruent corresponding parts—their matching sides and angles. When you name congruent polygons, you must list corresponding vertices in the same order.

Example



$ABCD \cong EFGH$

$\overline{AB} \cong \overline{EF}$ $\quad \overline{BC} \cong \overline{FG}$

$\overline{CD} \cong \overline{GH}$ $\quad \overline{DA} \cong \overline{HE}$

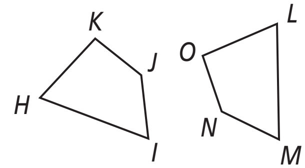
$\angle A \cong \angle E$ $\quad \angle B \cong \angle F$

$\angle C \cong \angle G$ $\quad \angle D \cong \angle H$

Plan

How do you know which sides and angles correspond?
The congruence statement *HIJK* ≅ *LMNO* tells you which parts correspond.

Problem 1 Finding Congruent Parts



If $HIJK \cong LMNO$, what are the congruent corresponding parts?

Sides: $\overline{HI} \cong \overline{LM}$ $\quad \overline{IJ} \cong \overline{MN}$ $\quad \overline{JK} \cong \overline{NO}$ $\quad \overline{KH} \cong \overline{OL}$

Angles: $\angle H \cong \angle L$ $\quad \angle I \cong \angle M$ $\quad \angle J \cong \angle N$ $\quad \angle K \cong \angle O$

Got It? **1.** If $\triangle WYS \cong \triangle MKV$, what are the congruent corresponding parts?

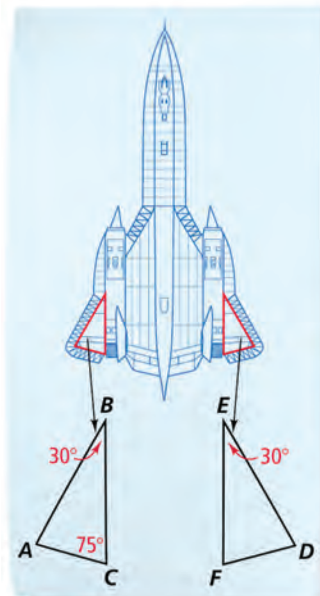
Plan

You know two angle measures in $\triangle ABC$. How can they help?
In the congruent triangles, $\angle D$ corresponds to $\angle A$, so you know that $\angle D \cong \angle A$. You can find $m\angle D$ by first finding $m\angle A$.

Problem 2 Using Congruent Parts

Multiple Choice **The wings of an SR-71 Blackbird aircraft suggest congruent triangles. What is $m\angle D$?**

Ⓐ 30 Ⓑ 75 Ⓒ 105 Ⓓ 150



Think	Write
Use the Triangle Angle-Sum Theorem to write an equation involving $m\angle A$.	$m\angle A + 30 + 75 = 180$
Solve for $m\angle A$.	$m\angle A + 105 = 180$ $m\angle A = 75$
$\angle A$ and $\angle D$ are corresponding parts of congruent triangles, so $\angle A \cong \angle D$.	$m\angle A = m\angle D = 75$ The correct answer is B.

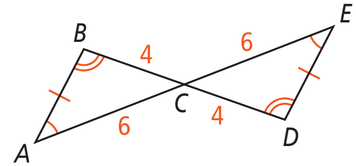
Got It? **2.** Suppose that $\triangle WYS \cong \triangle MKV$. If $m\angle W = 62$ and $m\angle Y = 35$, what is $m\angle V$? Explain.

Plan

How do you determine whether two triangles are congruent?
Compare each pair of corresponding parts. If all six pairs are congruent, then the triangles are congruent.

Problem 3 Finding Congruent Triangles

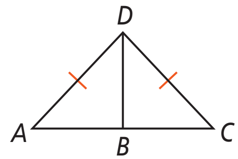
Are the triangles congruent? Justify your answer.



$\overline{AB} \cong \overline{ED}$	Given
$\overline{BC} \cong \overline{DC}$	$BC = 4 = DC$
$\overline{AC} \cong \overline{EC}$	$AC = 6 = EC$
$\angle A \cong \angle E, \angle B \cong \angle D$	Given
$\angle BCA \cong \angle DCE$	Vertical angles are congruent.

$\triangle ABC \cong \triangle EDC$ by the definition of congruent triangles.

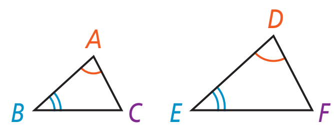
Got It? 3. Is $\triangle ABD \cong \triangle CBD$? Justify your answer.



Recall the Triangle Angle-Sum Theorem: The sum of the measures of the angles in a triangle is 180. The next theorem follows from the Triangle Angle-Sum Theorem.

take note Theorem 4-1 Third Angles Theorem

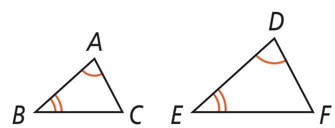
Theorem	If . . .	Then . . .
If two angles of one triangle are congruent to two angles of another triangle, then the third angles are congruent.	$\angle A \cong \angle D$ and $\angle B \cong \angle E$	$\angle C \cong \angle F$



Proof Proof of Theorem 4-1: Third Angles Theorem

Given: $\angle A \cong \angle D, \angle B \cong \angle E$

Prove: $\angle C \cong \angle F$



Statements	Reasons
1) $\angle A \cong \angle D, \angle B \cong \angle E$	**1)** Given
2) $m\angle A = m\angle D, m\angle B = m\angle E$	**2)** Def. of $\cong$ $\measuredangle$s
3) $m\angle A + m\angle B + m\angle C = 180$, $m\angle D + m\angle E + m\angle F = 180$	**3)** $\triangle$ Angle-Sum Thm.
4) $m\angle A + m\angle B + m\angle C = m\angle D + m\angle E + m\angle F$	**4)** Subst. Prop.
5) $m\angle D + m\angle E + m\angle C = m\angle D + m\angle E + m\angle F$	**5)** Subst. Prop.
6) $m\angle C = m\angle F$	**6)** Subtraction Prop. of =
7) $\angle C \cong \angle F$	**7)** Def. of $\cong$ $\measuredangle$s

Plan

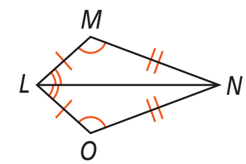
You know four pairs of congruent parts. What else do you need to prove the triangles congruent? You need a third pair of congruent sides and a third pair of congruent angles.

Proof

Problem 4 Proving Triangles Congruent

Given: $\overline{LM} \cong \overline{LO}$, $\overline{MN} \cong \overline{ON}$,
$\angle M \cong \angle O$, $\angle MLN \cong \angle OLN$

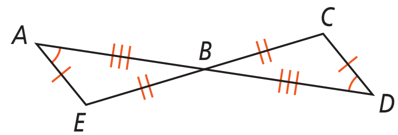
Prove: $\triangle LMN \cong \triangle LON$



Statements	Reasons
1) $\overline{LM} \cong \overline{LO}$, $\overline{MN} \cong \overline{ON}$	**1)** Given
2) $\overline{LN} \cong \overline{LN}$	**2)** Reflexive Property of $\cong$
3) $\angle M \cong \angle O$, $\angle MLN \cong \angle OLN$	**3)** Given
4) $\angle MNL \cong \angle ONL$	**4)** Third Angles Theorem
5) $\triangle LMN \cong \triangle LON$	**5)** Definition of $\cong$ triangles

Got It? **4. Given:** $\angle A \cong \angle D$, $\overline{AE} \cong \overline{DC}$,
$\overline{EB} \cong \overline{CB}$, $\overline{BA} \cong \overline{BD}$

Prove: $\triangle AEB \cong \triangle DCB$



Lesson Check

Do you know HOW?

Complete the following statements.

1. Given: $\triangle QXR \cong \triangle NYC$

a. $\overline{QX} \cong$ __?__

b. $\angle Y \cong$ __?__

2. Given: $\triangle BAT \cong \triangle FOR$

a. $\overline{TA} \cong$ __?__

b. $\angle R \cong$ __?__

3. Given: $BAND \cong LUCK$

a. $\angle U \cong$ __?__

b. $\overline{DB} \cong$ __?__

c. $NDBA \cong$ __?__

4. In $\triangle MAP$ and $\triangle TIE$, $\angle A \cong \angle I$ and $\angle P \cong \angle E$.

a. What is the relationship between $\angle M$ and $\angle T$?

b. If $m\angle A = 52$ and $m\angle P = 36$, what is $m\angle T$?

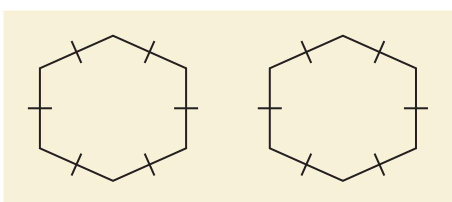
Do you UNDERSTAND?

MATHEMATICAL PRACTICES

5. Open-Ended When do you think you might need to know that things are congruent in your everyday life?

6. If each angle in one triangle is congruent to its corresponding angle in another triangle, are the two triangles congruent? Explain.

7. Error Analysis Walter sketched the diagram below. He claims it shows that the two polygons are congruent. What information is missing to support his claim?

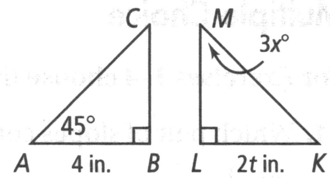


Name ______________________ Class ____________ Date ____________

4-1 Think About a Plan

Congruent Figures

Algebra Find the values of the variables.



$\triangle ABC \cong \triangle KLM$

Know

1. What do you know about the measure of each of the non-right angles?

____

2. What do you know about the length of each of the legs?

____

3. What types of triangles are shown in the figure?

____

Need

4. What information do you need to know to find the value of *x?*

____

5. What information do you need to know to find the value of *t?*

____

Plan

6. How can you find the value of *x?* What is its value?

____

7. How do you find the value of *t?* What is its value?

____

4-2 Triangle Congruence by SSS and SAS

Common Core State Standards

G-SRT.B.5 Use congruence criteria for triangles to solve problems and prove relationships in geometric figures.

MP 1, MP 3, MP 4, MP 7

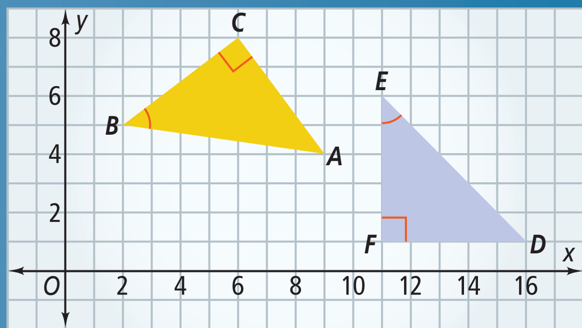
Objective To prove two triangles congruent using the SSS and SAS Postulates

How can you tell whether these triangles are congruent? In this lesson, you will learn the least amount of information required to tell if two triangles are congruent.

MATHEMATICAL PRACTICES

SOLVE IT! **Getting Ready!**

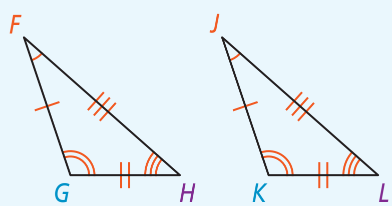
Are the triangles below congruent? How do you know?



In the Solve It, you looked for relationships between corresponding sides and angles. In Lesson 4-1, you learned that if two triangles have three pairs of congruent corresponding angles and three pairs of congruent corresponding sides, then the triangles are congruent.

If you know . . .

$\angle F \cong \angle J$	$\overline{FG} \cong \overline{JK}$
$\angle G \cong \angle K$	$\overline{GH} \cong \overline{KL}$
$\angle H \cong \angle L$	$\overline{FH} \cong \overline{JL}$



. . . then you know $\triangle FGH \cong \triangle JKL$.

However, this is more information about the corresponding parts than you need to prove triangles congruent.

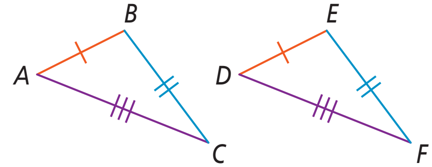
Essential Understanding You can prove that two triangles are congruent without having to show that *all* corresponding parts are congruent. In this lesson, you will prove triangles congruent by using (1) three pairs of corresponding sides and (2) two pairs of corresponding sides and one pair of corresponding angles.

take note

Postulate 4-1 Side-Side-Side (SSS) Postulate

Postulate
If the three sides of one triangle are congruent to the three sides of another triangle, then the two triangles are congruent.

If . . .
$\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, $\overline{AC} \cong \overline{DF}$



Then . . .
$\triangle ABC \cong \triangle DEF$

As described in Chapter 1, a postulate is an accepted statement of fact. The Side-Side-Side Postulate is perhaps the most logical fact about triangles. It agrees with the notion that triangles are rigid figures; their shape does not change until pressure on their sides forces them to break. This rigidity property is important to architects and engineers when they build things such as bicycle frames and steel bridges.

Proof

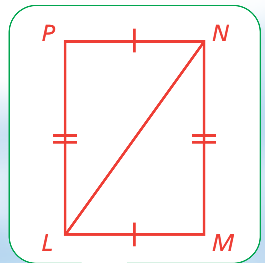
Problem 1 Using SSS

Given: $\overline{LM} \cong \overline{NP}$, $\overline{LP} \cong \overline{NM}$

Prove: $\triangle LMN \cong \triangle NPL$

Plan

You have two pairs of congruent sides. What else do you need?
You need a third pair of congruent corresponding sides. Notice that the triangles share a common side, $\overline{LN}$.



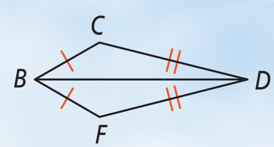
$\overline{LM} \cong \overline{NP}$
Given

$\overline{LN} \cong \overline{LN}$
Reflexive Prop. of $\cong$

$\overline{LP} \cong \overline{NM}$
Given

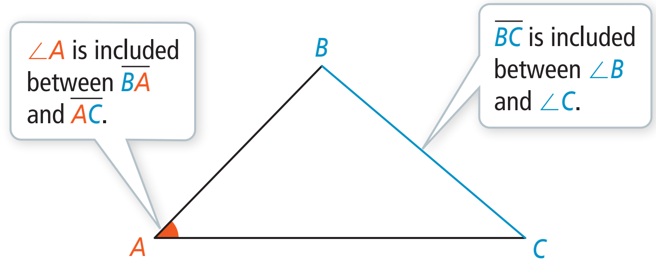
$\triangle LMN \cong \triangle NPL$
SSS

Got It? **1. Given:** $\overline{BC} \cong \overline{BF}$, $\overline{CD} \cong \overline{FD}$
Prove: $\triangle BCD \cong \triangle BFD$



You can also show relationships between a pair of corresponding sides and an *included* angle.

The word *included* refers to the angles and the sides of a triangle as shown at the right.



take note

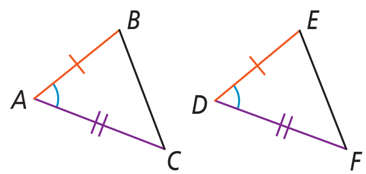
Postulate 4-2 Side-Angle-Side (SAS) Postulate

Postulate

If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the two triangles are congruent.

If . . .

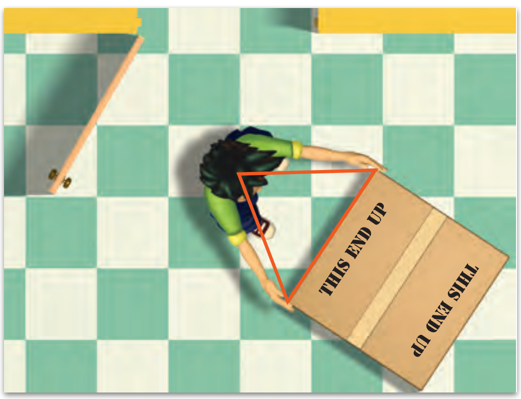
$\overline{AB} \cong \overline{DE}$, $\angle A \cong \angle D$, $\overline{AC} \cong \overline{DF}$



Then . . .

$\triangle ABC \cong \triangle DEF$

You likely have used the properties of the Side-Angle-Side Postulate before. For example, SAS can help you determine whether a box will fit through a doorway.



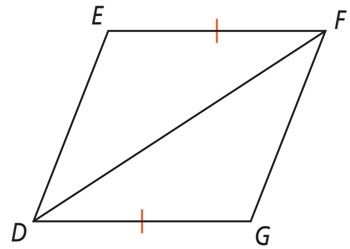
Suppose you keep your arms at a fixed angle as you move from the box to the doorway. The triangle you form with the box is congruent to the triangle you form with the doorway. The two triangles are congruent because two sides and the included angle of one triangle are congruent to the two sides and the included angle of the other triangle.

Problem 2 Using SAS

Plan

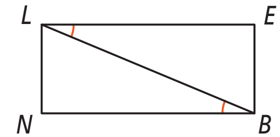
Do you need another pair of congruent sides?
Look at the diagram. The triangles share $\overline{DF}$. So, you already have two pairs of congruent sides.

What other information do you need to prove $\triangle DEF \cong \triangle FGD$ by SAS? Explain.

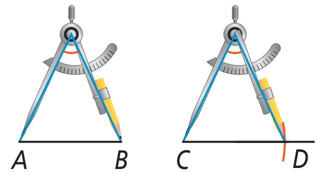


The diagram shows that $\overline{EF} \cong \overline{GD}$. Also, $\overline{DF} \cong \overline{DF}$ by the Reflexive Property of Congruence. To prove that $\triangle DEF \cong \triangle FGD$ by SAS, you must have congruent included angles. You need to know that $\angle EFD \cong \angle GDF$.

Got It? **2.** What other information do you need to prove $\triangle LEB \cong \triangle BNL$ by SAS?



Recall that, in Lesson 1-6, you learned to construct segments using a compass open to a fixed angle. Now you can show that it works. Similar to the situation with the box and the doorway, the Side-Angle-Side Postulate tells you that the triangles outlined at the right are congruent. So, $\overline{AB} \cong \overline{CD}$.



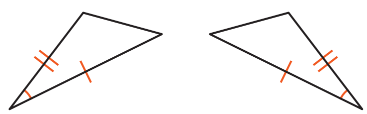
Problem 3 Identifying Congruent Triangles

Plan

What should you look for first, sides or angles?
Start with sides. If you have three pairs of congruent sides, use SSS. If you have two pairs of congruent sides, look for a pair of congruent included angles.

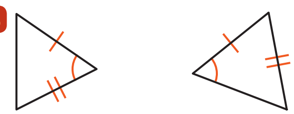
Would you use SSS or SAS to prove the triangles congruent? If there is not enough information to prove the triangles congruent by SSS or SAS, write *not enough information*. Explain your answer.

A



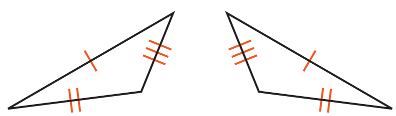
Use SAS because two pairs of corresponding sides and their included angles are congruent.

B



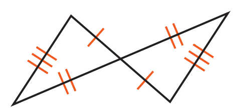
There is not enough information; two pairs of corresponding sides are congruent, but one of the angles is not the included angle.

C



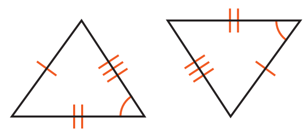
Use SSS because three pairs of corresponding sides are congruent.

D



Use SSS or SAS because all three pairs of corresponding sides and a pair of included angles (the vertical angles) are congruent.

Got It? **3.** Would you use SSS or SAS to prove the triangles at the right congruent? Explain.



Lesson Check

Do you know HOW?

1. In $\triangle PEN$, name the angle that is included between the given sides.

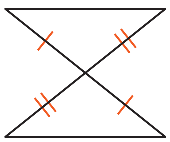
a. $\overline{PE}$ and $\overline{EN}$ **b.** $\overline{NP}$ and $\overline{PE}$

2. In $\triangle HAT$, between which sides is the given angle included?

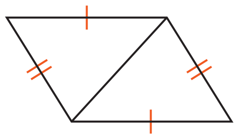
a. $\angle H$ **b.** $\angle T$

Name the postulate you would use to prove the triangles congruent.

3.



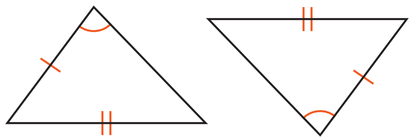
4.



Do you UNDERSTAND? MATHEMATICAL PRACTICES

5. Compare and Contrast How are the SSS Postulate and the SAS Postulate alike? How are they different?

6. Error Analysis Your friend thinks that the triangles shown below are congruent by SAS. Is your friend correct? Explain.



7. Reasoning A carpenter trims a triangular peak of a house with three 7-ft pieces of molding. The carpenter uses 21 ft of molding to trim a second triangular peak. Are the two triangles formed congruent? Explain.

Name ______________________ Class ____________ Date ____________

4-2 Think About a Plan

Triangle Congruence by SSS and SAS

Use the Distance Formula to determine whether $\triangle ABC$ and $\triangle DEF$ are congruent. Justify your answer.

$A(1, 4), B(5, 5), C(2, 2)$

$D(-5, 1), E(-1, 0), F(-4, 3)$

Understanding the Problem

1. You need to determine if $\triangle ABC \cong \triangle DEF$. What are the three ways you know to prove triangles congruent?

____

____

2. What information is given in the problem?

____

Planning the Solution

3. If you use the SSS Postulate to determine whether the triangles are congruent, what information do you need to find?

____

4. How can you find distances on a coordinate plane without measuring?

____

5. In an ordered pair, which number is the x-coordinate? Which is the y-coordinate?

____

Getting an Answer

6. Find the length of each segment using the Distance Formula, $D = \sqrt{(y_1 - y_2)^2 + (x_1 - x_2)^2}$. Your answers may be in simplest radical form.

$\overline{AB}$ $\boxed{}$ $\overline{BC}$ $\boxed{}$ $\overline{CA}$ $\boxed{}$

$\overline{DE}$ $\boxed{}$ $\overline{EF}$ $\boxed{}$ $\overline{FD}$ $\boxed{}$

7. Using the SSS Postulate, are the triangles congruent? Explain.

____

4-3 Triangle Congruence by ASA and AAS

Mathematics Florida Standards

MAFS.912.G-SRT.2.5 Use congruence ... criteria for triangles to solve problems and prove relationships in geometric figures.

Common Core State Standards

G-SRT.B.5 Use congruence ... criteria for triangles to solve problems and prove relationships in geometric figures.

MP 1, MP 3, MP 7

Objective To prove two triangles congruent using the ASA Postulate and the AAS Theorem

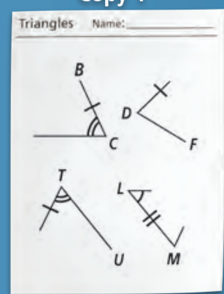
Use what you already know about proving triangles congruent. What is your plan for finding an answer?

MATHEMATICAL PRACTICES

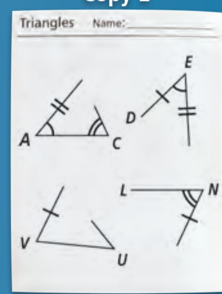
SOLVE IT! **Getting Ready!**

Oh no! The school's photocopier is not working correctly. The copies all have some ink missing. Below are two photocopies of the same geometry worksheet. Which triangles are congruent? How do you know?

Copy 1



Copy 2



You already know that triangles are congruent if two pairs of sides and the included angles are congruent (SAS). You can also prove triangles congruent using other groupings of angles and sides.

Essential Understanding You can prove that two triangles are congruent without having to show that *all* corresponding parts are congruent. In this lesson, you will prove triangles congruent by using one pair of corresponding sides and two pairs of corresponding angles.

take note

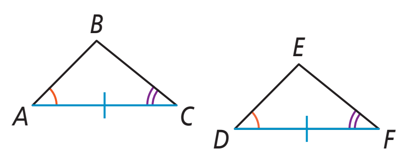
Postulate 4-3 Angle-Side-Angle (ASA) Postulate

Postulate

If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the two triangles are congruent.

If . . .

$\angle A \cong \angle D$, $\overline{AC} \cong \overline{DF}$, $\angle C \cong \angle F$

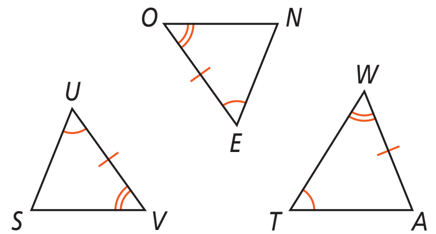


Then . . .

$\triangle ABC \cong \triangle DEF$

Problem 1 Using ASA

Which two triangles are congruent by ASA? Explain.



Know

From the diagram you know
- $\angle U \cong \angle E \cong \angle T$
- $\angle V \cong \angle O \cong \angle W$
- $\overline{UV} \cong \overline{EO} \cong \overline{AW}$

Need

To use ASA, you need two pairs of congruent angles and a pair of included congruent sides.

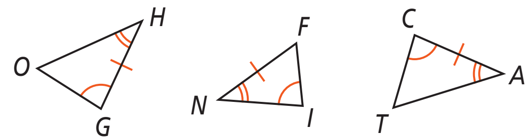
Plan

You already have pairs of congruent angles. So, identify the included side for each triangle and see whether it has a congruence marking.

In $\triangle SUV$, $\overline{UV}$ is included between $\angle U$ and $\angle V$ and has a congruence marking. In $\triangle NEO$, $\overline{EO}$ is included between $\angle E$ and $\angle O$ and has a congruence marking. In $\triangle ATW$, $\overline{TW}$ is included between $\angle T$ and $\angle W$ but does *not* have a congruence marking.

Since $\angle U \cong \angle E$, $\overline{UV} \cong \overline{EO}$, and $\angle V \cong \angle O$, $\triangle SUV \cong \triangle NEO$.

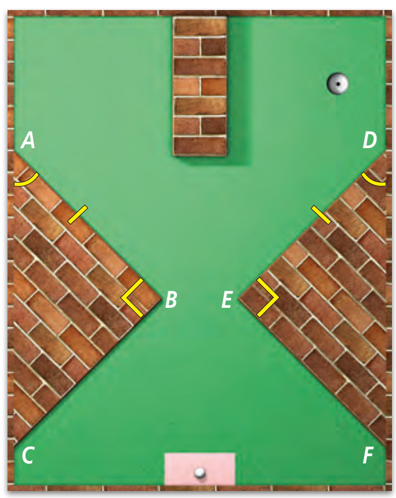
Got It? **1.** Which two triangles are congruent by ASA? Explain.



Proof

Problem 2 Writing a Proof Using ASA

Recreation **Members of a teen organization are building a miniature golf course at your town's youth center. The design plan calls for the first hole to have two congruent triangular bumpers. Prove that the bumpers on the first hole, shown at the right, meet the conditions of the plan.**



Given: $\overline{AB} \cong \overline{DE}$, $\angle A \cong \angle D$, $\angle B$ and $\angle E$ are right angles

Prove: $\triangle ABC \cong \triangle DEF$

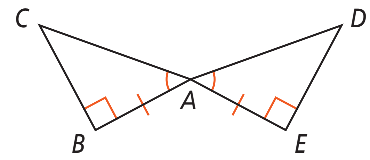
Proof: $\angle B \cong \angle E$ because all right angles are congruent, and you are given that $\angle A \cong \angle D$. $\overline{AB}$ and $\overline{DE}$ are included sides between the two pairs of congruent angles. You are given that $\overline{AB} \cong \overline{DE}$. Thus, $\triangle ABC \cong \triangle DEF$ by ASA.

Plan

Can you use a plan similar to the plan in Problem 1?
Yes. Use the diagram to identify the included side for the marked angles in each triangle.

Got It? **2. Given:** $\angle CAB \cong \angle DAE$, $\overline{BA} \cong \overline{EA}$, $\angle B$ and $\angle E$ are right angles

Prove: $\triangle ABC \cong \triangle AED$



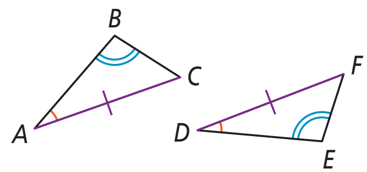
You can also prove triangles congruent by using two angles and a nonincluded side, as stated in the theorem below.

take note

Theorem 4-2 Angle-Angle-Side (AAS) Theorem

Theorem
If two angles and a nonincluded side of one triangle are congruent to two angles and the corresponding nonincluded side of another triangle, then the triangles are congruent.

If . . .
$\angle A \cong \angle D$, $\angle B \cong \angle E$, $\overline{AC} \cong \overline{DF}$

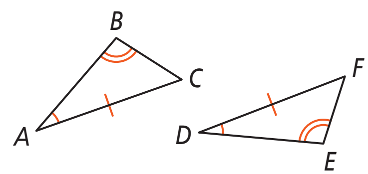


Then . . .
$\triangle ABC \cong \triangle DEF$

Proof Proof of Theorem 4-2: Angle-Angle-Side Theorem

Given: $\angle A \cong \angle D$, $\angle B \cong \angle E$, $\overline{AC} \cong \overline{DF}$

Prove: $\triangle ABC \cong \triangle DEF$



$\angle A \cong \angle D$
Given

$\angle B \cong \angle E$
Given

$\overline{AC} \cong \overline{DF}$
Given

$\angle C \cong \angle F$
Third Angles Theorem

$\triangle ABC \cong \triangle DEF$
ASA

You have seen and used three methods of proof in this book—two-column, paragraph, and flow proof. Each method is equally as valid as the others. Unless told otherwise, you can choose any of the three methods to write a proof. Just be sure your proof always presents logical reasoning with justification.

Plan

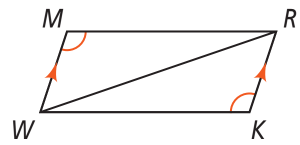
How does information about parallel sides help?
You will need another pair of congruent angles to use AAS. Think back to what you learned in Chapter 3. $\overline{WR}$ is a transversal here.

Proof

Problem 3 Writing a Proof Using AAS

Given: $\angle M \cong \angle K$, $\overline{WM} \parallel \overline{RK}$

Prove: $\triangle WMR \cong \triangle RKW$

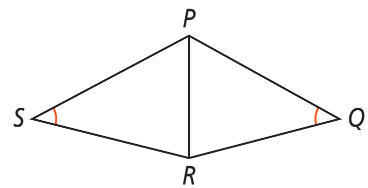


Statements	Reasons
1) $\angle M \cong \angle K$	**1)** Given
2) $\overline{WM} \parallel \overline{RK}$	**2)** Given
3) $\angle MWR \cong \angle KRW$	**3)** If lines are $\parallel$, then alternate interior $\angle$s are $\cong$.
4) $\overline{WR} \cong \overline{WR}$	**4)** Reflexive Property of Congruence
5) $\triangle WMR \cong \triangle RKW$	**5)** AAS

Got It? **3. a. Given:** $\angle S \cong \angle Q$, $\overline{RP}$ bisects $\angle SRQ$

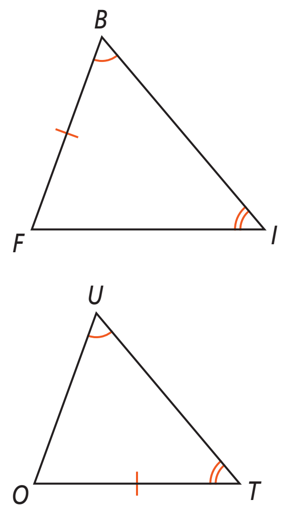
Prove: $\triangle SRP \cong \triangle QRP$

b. Reasoning In Problem 3, how could you prove that $\triangle WMR \cong \triangle RKW$ by ASA? Explain.



Problem 4 Determining Whether Triangles Are Congruent

Multiple Choice Use the diagram at the right. Which of the following statements best represents the answer and justification to the question, "Is $\triangle BIF \cong \triangle UTO$?"



(A) Yes, the triangles are congruent by ASA.

(B) No, $\overline{FB}$ and $\overline{OT}$ are not corresponding sides.

(C) Yes, the triangles are congruent by AAS.

(D) No, $\angle B$ and $\angle U$ are not corresponding angles.

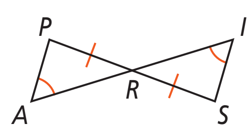
Think

Can you eliminate any of the choices?
Yes. If $\triangle BIF \cong \triangle UTO$ then $\angle B$ and $\angle U$ would be corresponding angles. You can eliminate choice D.

The diagram shows that two pairs of angles and one pair of sides are congruent. The third pair of angles is congruent by the Third Angles Theorem. To prove these triangles congruent, you need to satisfy ASA or AAS.

ASA and AAS both fail because $\overline{FB}$ and $\overline{TO}$ are not included between the same pair of congruent corresponding angles, so they are not corresponding sides. The triangles are not necessarily congruent. The correct answer is B.

Got It? **4.** Are $\triangle PAR$ and $\triangle SIR$ congruent? Explain.



Lesson Check

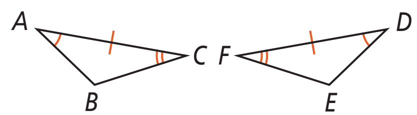
Do you know HOW?

1. In $\triangle RST$, which side is included between $\angle R$ and $\angle S$?

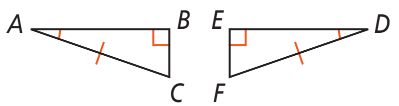
2. In $\triangle NOM$, $\overline{NO}$ is included between which angles?

Which postulate or theorem could you use to prove $\triangle ABC \cong \triangle DEF$?

3.



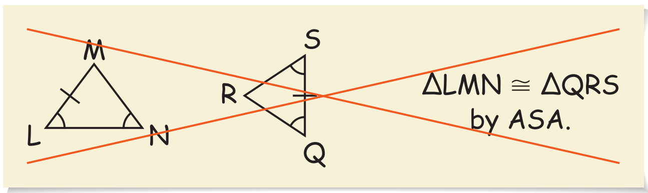
4.



Do you UNDERSTAND? MATHEMATICAL PRACTICES

5. Compare and Contrast How are the ASA Postulate and the SAS Postulate alike? How are they different?

6. Error Analysis Your friend asks you for help on a geometry exercise. Below is your friend's paper. What error did your friend make? Explain.



7. Reasoning Suppose $\angle E \cong \angle I$ and $\overline{FE} \cong \overline{GI}$. What else must you know in order to prove $\triangle FDE \cong \triangle GHI$ by ASA? By AAS?

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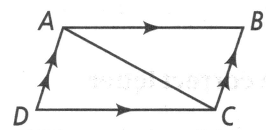
4-3 Think About a Plan

Triangle Congruence by ASA and AAS

Given: $\overline{AB} \parallel \overline{CD}$, $\overline{AD} \parallel \overline{CB}$

Prove: $\triangle ABC \cong \triangle CDA$



1. What do you need to find to solve the problem?

____

____

2. What are the corresponding parts of the two triangles?

____

____

3. What word would you use to describe $\overline{AC}$? ______________________

4. What can you show about angles in the triangles that can indicate congruency?

____

5. What do you know about a side or sides of the triangles that can be used to show congruency?

____

____

6. Write a proof in paragraph form.

____

____

____

____

4-4 Using Corresponding Parts of Congruent Triangles

Mathematics Florida Standards

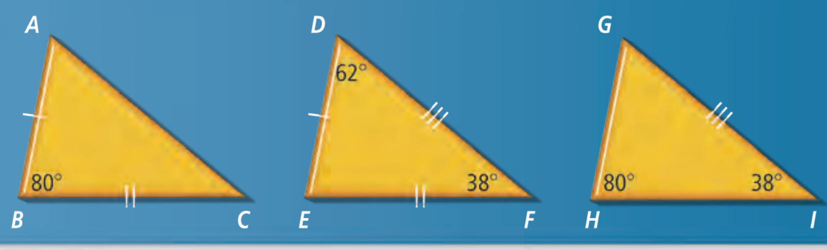
MAFS.912.G-SRT.2.5 Use congruence . . . criteria for triangles to solve problems and to prove relationships in geometric figures. **Also MAFS.912.G-SRT.4.12**

MP 1, MP 3

Objective To use triangle congruence and corresponding parts of congruent triangles to prove that parts of two triangles are congruent

SOLVE IT! Getting Ready!

Is △ABC congruent to △GHI? How do you know?



How does △DEF help you solve this problem?

MATHEMATICAL PRACTICES

With SSS, SAS, ASA, and AAS, you know how to use three congruent parts of two triangles to show that the triangles are congruent. Once you know that two triangles are congruent, you can make conclusions about their other corresponding parts because, by definition, corresponding parts of congruent triangles are congruent.

Essential Understanding If you know two triangles are congruent, then you know that every pair of their corresponding parts is also congruent.

Problem 1 Proving Parts of Triangles Congruent

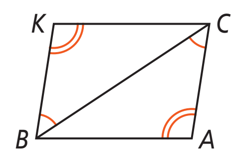
Proof

Think

In the diagram, which congruent pair is not marked?
The third angles of both triangles are congruent. But there is no AAA congruence rule. So, find a congruent pair of sides.

Given: $\angle KBC \cong \angle ACB$, $\angle K \cong \angle A$

Prove: $\overline{KB} \cong \overline{AC}$



$\angle KBC \cong \angle ACB$
Given

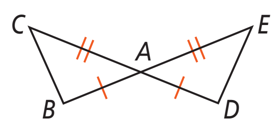
$\overline{BC} \cong \overline{BC}$
Reflexive Property of ≅

$\angle K \cong \angle A$
Given

$\triangle KBC \cong \triangle ACB$
AAS Theorem

$\overline{KB} \cong \overline{AC}$
Corresp. parts of ≅ △ are ≅.

Got It? **1. Given:** $\overline{BA} \cong \overline{DA}$, $\overline{CA} \cong \overline{EA}$
Prove: $\angle C \cong \angle E$



Proof **Problem 2** **Proving Triangle Parts Congruent to Measure Distance** STEM

Plan

Which congruency rule can you use? You have information about two pairs of angles. *Guess-and-check* AAS and ASA.

Measurement Thales, a Greek philosopher, is said to have developed a method to measure the distance to a ship at sea. He made a compass by nailing two sticks together. Standing on top of a tower, he would hold one stick vertical and tilt the other until he could see the ship S along the line of the tilted stick. With this compass setting, he would find a landmark L on the shore along the line of the tilted stick. How far would the ship be from the base of the tower?

Given: $\angle TRS$ and $\angle TRL$ are right angles, $\angle RTS \cong \angle RTL$

Prove: $\overline{RS} \cong \overline{RL}$

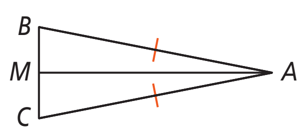
Statements	Reasons
1) $\angle RTS \cong \angle RTL$	**1)** Given
2) $\overline{TR} \cong \overline{TR}$	**2)** Reflexive Property of Congruence
3) $\angle TRS$ and $\angle TRL$ are right angles.	**3)** Given
4) $\angle TRS \cong \angle TRL$	**4)** All right angles are congruent.
5) $\triangle TRS \cong \triangle TRL$	**5)** ASA Postulate
6) $\overline{RS} \cong \overline{RL}$	**6)** Corresponding parts of $\cong$ $\triangle$s are $\cong$.

The distance between the ship and the base of the tower would be the same as the distance between the base of the tower and the landmark.

Got It? **2. a. Given:** $\overline{AB} \cong \overline{AC}$, M is the midpoint of $\overline{BC}$

Prove: $\angle AMB \cong \angle AMC$

b. Reasoning If the landmark were not at sea level, would the method in Problem 2 work? Explain.

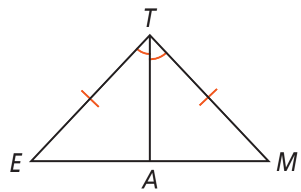


Lesson Check

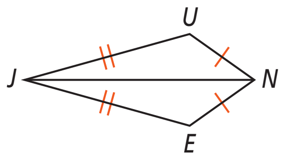
Do you know HOW?

Name the postulate or theorem that you can use to show the triangles are congruent. Then explain why the statement is true.

1. $\overline{EA} \cong \overline{MA}$



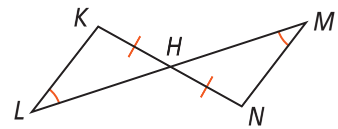
2. $\angle U \cong \angle E$



Do you UNDERSTAND? MATHEMATICAL PRACTICES

3. Reasoning How does the fact that corresponding parts of congruent triangles are congruent relate to the definition of congruent triangles?

4. Error Analysis Find and correct the error(s) in the proof.



Given: $\overline{KH} \cong \overline{NH}$, $\angle L \cong \angle M$

Prove: H is the midpoint of $\overline{LM}$.

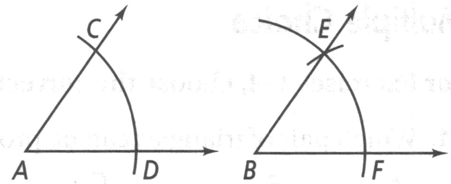
Proof: $\overline{KH} \cong \overline{NH}$ because it is given. $\angle L \cong \angle M$ because it is given. $\angle KHL \cong \angle NHM$ because vertical angles are congruent. So, $\triangle KHL \cong \triangle MHN$ by ASA Postulate. Since corresponding parts of congruent triangles are congruent, $\overline{LH} \cong \overline{MH}$. By the definition of midpoint, H is the midpoint of $\overline{LM}$.

Name ______________________ Class ____________ Date ____________

4-4 Think About a Plan

Using Corresponding Parts of Congruent Triangles

Constructions The construction of $\angle B$ congruent to given $\angle A$ is shown. $\overline{AD} \cong \overline{BF}$ because they are the radii of the same circle. $\overline{DC} \cong \overline{FE}$ because both arcs have the same compass settings. Explain why you can conclude that $\angle A \cong \angle B$.



Understanding the Problem

1. What is the problem asking you to prove?

2. Segments $\overline{DC}$ and $\overline{FE}$ are not drawn on the construction. Draw them in. What figures are formed by drawing these segments?

3. What information do you need to be able to use corresponding parts of congruent triangles?

Planning the Solution

4. To use corresponding parts of congruent triangles, which two triangles do you need to show to be congruent?

5. What reason can you use to state that $\overline{AC} \cong \overline{BE}$?

Getting an Answer

6. Write a paragraph proof that uses corresponding parts of congruent triangles to prove that $\angle A \cong \angle B$.

4-5 Isosceles and Equilateral Triangles

Mathematics Florida Standards

MAFS.912.G-CO.3.10 Prove theorems about triangles . . . base angles of isosceles triangles are congruent . . . **Also MAFS.912.G-CO.4.13, MAFS.912.G-SRT.2.5**

MP 1, MP 3, MP 4

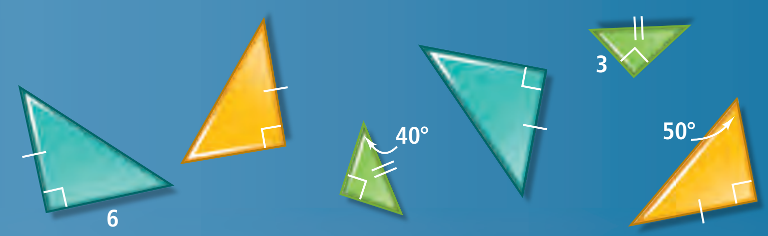
Objective To use and apply properties of isosceles and equilateral triangles

Solving puzzles is fun! Work with the pieces until they make a whole triangle. Look for patterns in your solution.

MATHEMATICAL PRACTICES

SOLVE IT! Getting Ready!

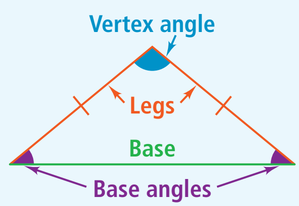
The triangles of the same color are congruent. Arrange the triangles to form one large triangle. You must use all the pieces. Make a sketch of this triangle. Classify this triangle by its sides. What are the angle measures of this triangle? Explain.



In the Solve It, you classified a triangle based on the lengths of its sides. You can also identify certain triangles based on information about their angles. In this lesson, you will learn how to use and apply properties of isosceles and equilateral triangles.

Essential Understanding The angles and sides of isosceles and equilateral triangles have special relationships.

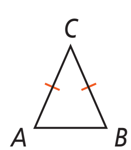
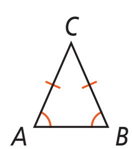
Isosceles triangles are common in the real world. You can frequently see them in structures such as bridges and buildings, as well as in art and design. The congruent sides of an isosceles triangle are its **legs.** The third side is the **base.** The two congruent legs form the **vertex angle.** The other two angles are the **base angles.**



Lesson Vocabulary
- legs of an isosceles triangle
- base of an isosceles triangle
- vertex angle of an isosceles triangle
- base angles of an isosceles triangle
- corollary

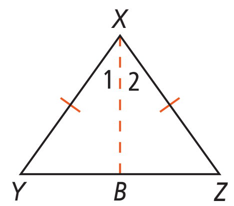
take note

Theorem 4-3 Isosceles Triangle Theorem

Theorem	If . . .	Then . . .
If two sides of a triangle are congruent, then the angles opposite those sides are congruent.	$\overline{AC} \cong \overline{BC}$ 	$\angle A \cong \angle B$ 

The proof of the Isosceles Triangle Theorem requires an auxiliary line.

Proof Proof of Theorem 4-3: Isosceles Triangle Theorem



Begin with isosceles $\triangle XYZ$ with $\overline{XY} \cong \overline{XZ}$. Draw $\overline{XB}$, the bisector of the vertex angle $\angle YXZ$.

Given: $\overline{XY} \cong \overline{XZ}$, $\overline{XB}$ bisects $\angle YXZ$

Prove: $\angle Y \cong \angle Z$

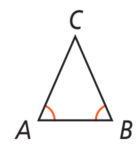
Proof: $\overline{XY} \cong \overline{XZ}$ is given. By the definition of angle bisector, $\angle 1 \cong \angle 2$. By the Reflexive Property of Congruence, $\overline{XB} \cong \overline{XB}$. So by the SAS Postulate, $\triangle XYB \cong \triangle XZB$. $\angle Y \cong \angle Z$ since corresponding parts of congruent triangles are congruent.

take note

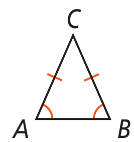
Theorem 4-4 Converse of the Isosceles Triangle Theorem

Theorem
If two angles of a triangle are congruent, then the sides opposite those angles are congruent.

If . . .
$\angle A \cong \angle B$



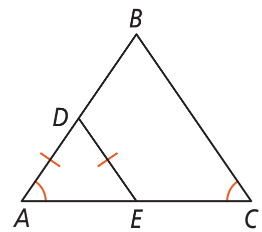
Then . . .
$\overline{AC} \cong \overline{BC}$



You will prove Theorem 4-4 in Exercise 23.

Problem 1 Using the Isosceles Triangle Theorems

Think
What are you looking for in the diagram?
To use the Isosceles Triangle Theorems, you need a pair of congruent angles or a pair of congruent sides.



A Is $\overline{AB}$ congruent to $\overline{CB}$? Explain.

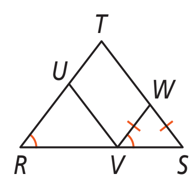
Yes. Since $\angle C \cong \angle A$, $\overline{AB} \cong \overline{CB}$ by the Converse of the Isosceles Triangle Theorem.

B Is $\angle A$ congruent to $\angle DEA$? Explain.

Yes. Since $\overline{AD} \cong \overline{ED}$, $\angle A \cong \angle DEA$ by the Isosceles Triangle Theorem.

Got It? 1. a. Is $\angle WVS$ congruent to $\angle S$? Is $\overline{TR}$ congruent to $\overline{TS}$? Explain.

b. Reasoning Can you conclude that $\triangle RUV$ is isosceles? Explain.

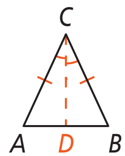


An isosceles triangle has a certain type of symmetry about a line through its vertex angle. The theorems in this lesson suggest this symmetry, which you will study in greater detail in Lesson 9-4.

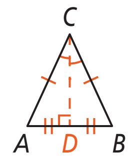
take note — Theorem 4-5

Theorem
If a line bisects the vertex angle of an isosceles triangle, then the line is also the perpendicular bisector of the base.

If . . .
$\overline{AC} \cong \overline{BC}$ and $\angle ACD \cong \angle BCD$



Then . . .
$\overline{CD} \perp \overline{AB}$ and $\overline{AD} \cong \overline{BD}$

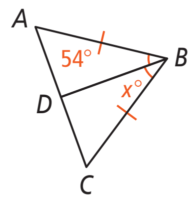


You will prove Theorem 4-5 in Exercise 26.

Problem 2 Using Algebra

GRIDDED RESPONSE

What is the value of x?

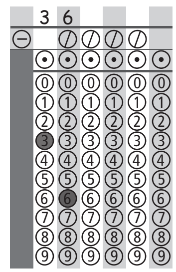


Think
What does the diagram tell you?
Since $\overline{AB} \cong \overline{CB}$, $\triangle ABC$ is isosceles. Since $\angle ABD \cong \angle CBD$, $\overline{BD}$ bisects the vertex angle of the isosceles triangle.

Since $\overline{AB} \cong \overline{CB}$, by the Isosceles Triangle Theorem, $\angle A \cong \angle C$. So $m\angle C = 54$.

Since $\overline{BD}$ bisects $\angle ABC$, you know by Theorem 4-5 that $\overline{BD} \perp \overline{AC}$. So $m\angle BDC = 90$.

$m\angle C + m\angle BDC + m\angle DBC = 180$	Triangle Angle-Sum Theorem
$54 + 90 + x = 180$	Substitute.
$x = 36$	Subtract 144 from each side.



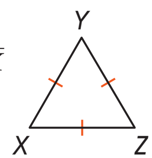
Got It? **2.** Suppose $m\angle A = 27$. What is the value of x?

A **corollary** is a theorem that can be proved easily using another theorem. Since a corollary is a theorem, you can use it as a reason in a proof.

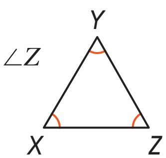
take note — Corollary to Theorem 4-3

Corollary
If a triangle is equilateral, then the triangle is equiangular.

If . . .
$\overline{XY} \cong \overline{YZ} \cong \overline{ZX}$



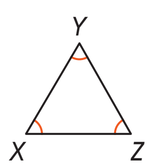
Then . . .
$\angle X \cong \angle Y \cong \angle Z$



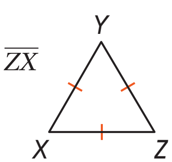
Corollary to Theorem 4-4

Corollary
If a triangle is equiangular, then the triangle is equilateral.

If . . .
$\angle X \cong \angle Y \cong \angle Z$



Then . . .
$\overline{XY} \cong \overline{YZ} \cong \overline{ZX}$



Problem 3 Finding Angle Measures

Design **What are the measures of $\angle A$, $\angle B$, and $\angle ADC$ in the photo at the right?**

Think	Write
The triangles are equilateral, so they are also equiangular. Find the measure of each angle of an equilateral triangle.	Let a = measure of one angle. $3a = 180$ $a = 60$
$\angle A$ and $\angle B$ are both angles in an equilateral triangle.	$m\angle A = m\angle B = 60$
Use the Angle Addition Postulate to find the measure of $\angle ADC$.	$m\angle ADC = m\angle ADE + m\angle CDE$
Both $\angle ADE$ and $\angle CDE$ are angles in an equilateral triangle. So $m\angle ADE = 60$ and $m\angle CDE = 60$. Substitute into the above equation and simplify.	$m\angle ADC = 60 + 60$ $m\angle ADC = 120$

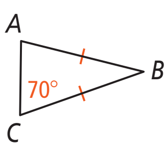
Got It? **3.** Suppose the triangles in Problem 3 are isosceles triangles, where $\angle ADE$, $\angle DEC$, and $\angle ECB$ are vertex angles. If the vertex angles each have a measure of 58, what are $m\angle A$ and $m\angle BCD$?

Lesson Check

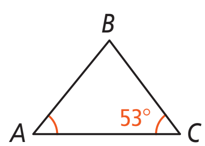
Do you know HOW?

1. What is $m\angle A$?

a.

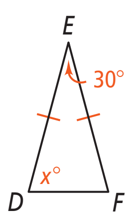


b.

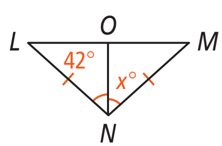


2. What is the value of x?

a.



b.



3. The measure of one base angle of an isosceles triangle is 23. What are the measures of the other two angles?

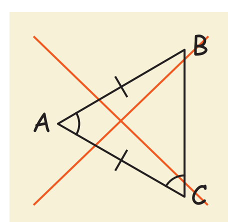
Do you UNDERSTAND? MATHEMATICAL PRACTICES

4. What is the relationship between sides and angles for each type of triangle?

a. isosceles

b. equilateral

5. **Error Analysis** Claudia drew an isosceles triangle. She asked Sue to mark it. Explain why the marking of the diagram is incorrect.



Algebra Review

Use With Lesson 4-6

Systems of Linear Equations

Common Core State Standards

Reviews A-REI.C.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Mathematics Florida Standards

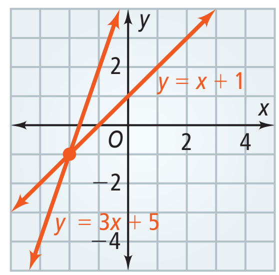
Reviews MAFS.912.A-REI.3.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

You can solve a system of equations in two variables by using substitution.

Example 1

Algebra **Solve the system.** $\quad y = 3x + 5$

$\qquad\qquad\qquad\qquad\quad y = x + 1$



$y = x + 1$	Start with one equation.
$3x + 5 = x + 1$	Substitute $3x + 5$ for y.
$2x = -4$	Solve for x.
$x = -2$	

Substitute -2 for x in either equation and solve for y.

$y = x + 1 = (-2) + 1 = -1$

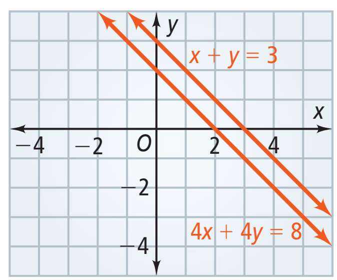
Since $x = -2$ and $y = 1$, the solution is $(-2, -1)$. This is the point of intersection of the two lines.

The graph of a linear system with *infinitely many solutions* is one line, and the graph of a linear system with *no solution* is two parallel lines.

Example 2

Algebra **Solve the system.** $\quad x + y = 3$

$\qquad\qquad\qquad\qquad\quad 4x + 4y = 8$



$x + y = 3$	Start with one equation.
$x = 3 - y$	Solve the equation for x.
$4(3 - y) + 4y = 8$	Substitute $3 - y$ for x in the second equation.
$12 - 4y + 4y = 8$	Solve for y.
$12 = 8$	False!

Since $12 = 8$ is a false statement, the system has no solution.

Exercises

Solve each system of equations.

1. $y = x - 4$
$y = 3x + 2$

2. $2x - y = 8$
$x + 2y = 9$

3. $3x + y = 4$
$-6x - 2y = 12$

4. $2x - 3 = y + 3$
$2x + y = -3$

5. $y = x + 1$
$x = y - 1$

6. $x - y = 4$
$3x - 3y = 6$

7. $y = -x + 2$
$2y = 4 - 2x$

8. $y = 2x - 1$
$y = 3x - 7$

Name ____________________ Class ____________ Date ____________

4-5 Think About a Plan

Isosceles and Equilateral Triangles

Algebra The length of the base of an isosceles triangle is x. The length of a leg is $2x - 5$. The perimeter of the triangle is 20. Find x.

Know

1. What is the perimeter of a triangle?

____

2. What is an isosceles triangle?

____

Need

3. What are the sides of an isosceles triangle called?

____

4. How many of each type of side are there?

____

5. The lengths of the base and one leg are given. What is the third side of the triangle called?

____

Plan

6. Write an expression for the length of the third side.

7. Write an equation for the perimeter of this isosceles triangle.

8. Solve the equation for x. Show your work.

4-6 Congruence in Right Triangles

Common Core State Standards

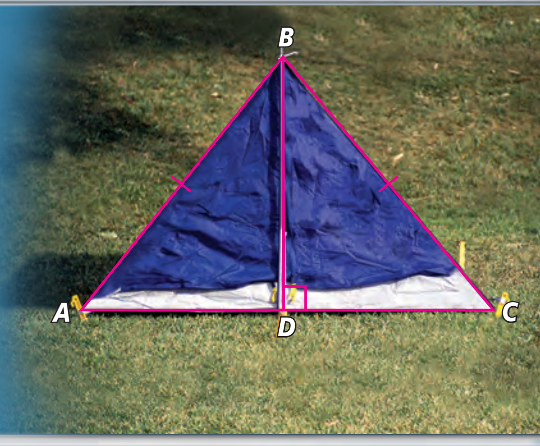
G-SRT.B.5 Use congruence criteria to solve problems and prove relationships in geometric figures.

MP 1, MP 3

Objective To prove right triangles congruent using the Hypotenuse-Leg Theorem

SOLVE IT! **Getting Ready!**

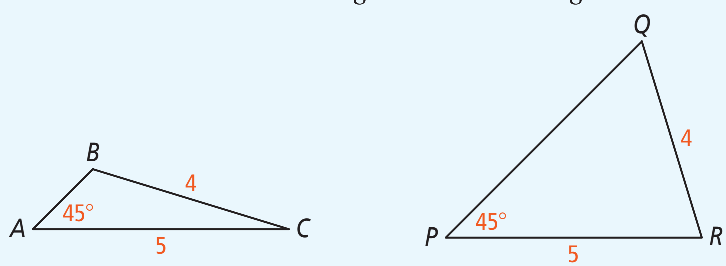
One of the tent flaps was damaged in a storm. Can you use the other flap as a pattern to replace it? Explain.



What does the large triangle tell you about angles in the figure?

MATHEMATICAL PRACTICES

In the diagram below, two sides and a nonincluded angle of one triangle are congruent to two sides and the nonincluded angle of another triangle.

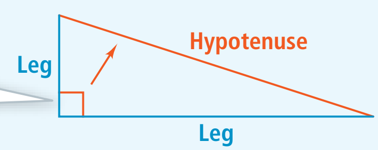


Lesson Vocabulary
- hypotenuse
- legs of a right triangle

Notice that the triangles are not congruent. So, you can conclude that Side-Side-Angle is *not* a valid method for proving two triangles congruent. This method, however, works in the special case of right triangles, where the right angles are the nonincluded angles.

In a right triangle, the side opposite the right angle is called the **hypotenuse.** It is the longest side in the triangle. The other two sides are called **legs.**

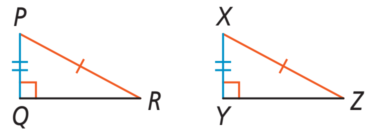
The right angle always "points" to the hypotenuse.



Essential Understanding You can prove that two triangles are congruent without having to show that *all* corresponding parts are congruent. In this lesson, you will prove right triangles congruent by using one pair of right angles, a pair of hypotenuses, and a pair of legs.

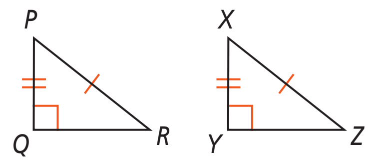
take note

Theorem 4-6 Hypotenuse-Leg (HL) Theorem

Theorem	If . . .	Then . . .
If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and a leg of another right triangle, then the triangles are congruent.	$\triangle PQR$ and $\triangle XYZ$ are right $\triangle$s, $\overline{PR} \cong \overline{XZ}$, and $\overline{PQ} \cong \overline{XY}$ 	$\triangle PQR \cong \triangle XYZ$

To prove the HL Theorem you will need to draw auxiliary lines to make a third triangle.

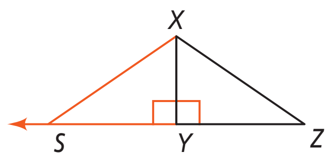
Proof Proof of Theorem 4-6: Hypotenuse-Leg Theorem



Given: $\triangle PQR$ and $\triangle XYZ$ are right triangles, with right angles Q and Y. $\overline{PR} \cong \overline{XZ}$ and $\overline{PQ} \cong \overline{XY}$.

Prove: $\triangle PQR \cong \triangle XYZ$

Proof: On $\triangle XYZ$, draw $\overrightarrow{ZY}$.



Mark point S so that $YS = QR$. Then, $\triangle PQR \cong \triangle XYS$ by SAS.

Since corresponding parts of congruent triangles are congruent, $\overline{PR} \cong \overline{XS}$. It is given that $\overline{PR} \cong \overline{XZ}$, so $\overline{XS} \cong \overline{XZ}$ by the Transitive Property of Congruence. By the Isosceles Triangle Theorem, $\angle S \cong \angle Z$, so $\triangle XYS \cong \triangle XYZ$ by AAS. Therefore, $\triangle PQR \cong \triangle XYZ$ by the Transitive Property of Congruence.

take note

Key Concept Conditions for HL Theorem

To use the HL Theorem, the triangles must meet three conditions.

Conditions

- There are two right triangles.
- The triangles have congruent hypotenuses.
- There is one pair of congruent legs.

Proof **Problem 1** **Using the HL Theorem**

On the basketball backboard brackets shown below, $\angle ADC$ and $\angle BDC$ are right angles and $\overline{AC} \cong \overline{BC}$. Are $\triangle ADC$ and $\triangle BDC$ congruent? Explain.

Plan

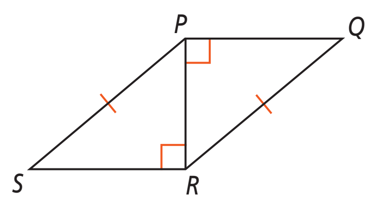
How can you visualize the two right triangles?
Imagine cutting $\triangle ABC$ along $\overline{DC}$. On either side of the cut, you get triangles with the same leg $\overline{DC}$.

- You are given that $\angle ADC$ and $\angle BDC$ are right angles. So, $\triangle ADC$ and $\triangle BDC$ are right triangles.
- The hypotenuses of the two right triangles are $\overline{AC}$ and $\overline{BC}$. You are given that $\overline{AC} \cong \overline{BC}$.
- $\overline{DC}$ is a common leg of both $\triangle ADC$ and $\triangle BDC$. $\overline{DC} \cong \overline{DC}$ by the Reflexive Property of Congruence.

Yes, $\triangle ADC \cong \triangle BDC$ by the HL Theorem.

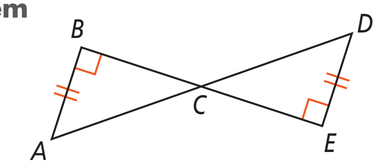
Got It? **1. a. Given:** $\angle PRS$ and $\angle RPQ$ are right angles, $\overline{SP} \cong \overline{QR}$

Prove: $\triangle PRS \cong \triangle RPQ$



b. Reasoning Your friend says, "Suppose you have two right triangles with congruent hypotenuses and one pair of congruent legs. It does not matter which leg in the first triangle is congruent to which leg in the second triangle. The triangles will be congruent." Is your friend correct? Explain.

Proof — Problem 2 Writing a Proof Using the HL Theorem



Given: $\overline{BE}$ bisects $\overline{AD}$ at C,
$\overline{AB} \perp \overline{BC}$, $\overline{DE} \perp \overline{EC}$, $\overline{AB} \cong \overline{DE}$

Prove: $\triangle ABC \cong \triangle DEC$

Plan

How can you get started?
Identify the hypotenuse of each right triangle. Prove that the hypotenuses are congruent.

$\overline{BE}$ bisects $\overline{AD}$. → $\overline{AC} \cong \overline{DC}$
Given → Def. of bisector

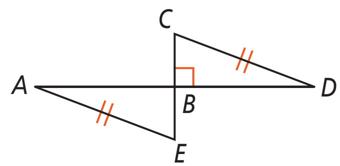
$\overline{AB} \perp \overline{BC}$, $\overline{DE} \perp \overline{EC}$ → $\angle ABC$ and $\angle DEC$ are right ⦜s. → $\triangle ABC$ and $\triangle DEC$ are right ⊿s.
Given → Def. of $\perp$ lines → Def. of right triangle

$\overline{AB} \cong \overline{DE}$
Given

$\triangle ABC \cong \triangle DEC$
HL Theorem

Got It? 2. Given: $\overline{CD} \cong \overline{EA}$, $\overline{AD}$ is the perpendicular bisector of $\overline{CE}$

Prove: $\triangle CBD \cong \triangle EBA$

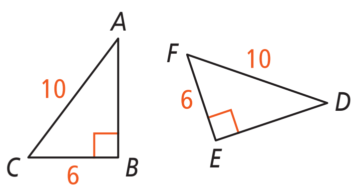


Lesson Check

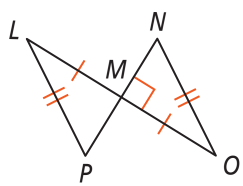
Do you know HOW?

Are the two triangles congruent? If so, write the congruence statement.

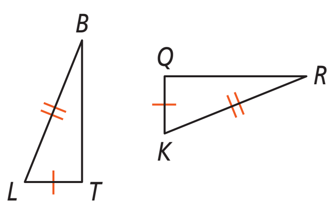
1.



2.



3.



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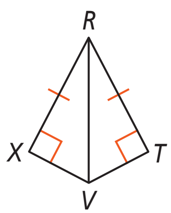
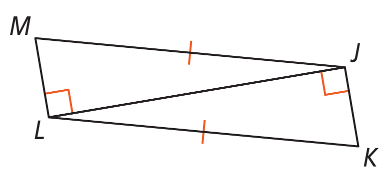


Do you UNDERSTAND? MATHEMATICAL PRACTICES

5. Vocabulary A right triangle has side lengths of 5 cm, 12 cm, and 13 cm. What is the length of the hypotenuse? How do you know?

6. Compare and Contrast How do the HL Theorem and the SAS Postulate compare? How are they different? Explain.

7. Error Analysis Your classmate says that there is not enough information to determine whether the two triangles below are congruent. Is your classmate correct? Explain.

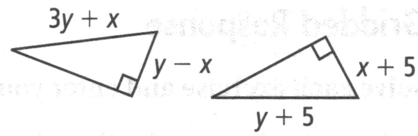


Name ____________________ Class ____________ Date ____________

4-6 Think About a Plan

Congruence in Right Triangles

Algebra For what values of x and y are the triangles congruent by HL?



Know

1. For two triangles to be congruent by the Hypotenuse-Leg Theorem, there must be a ____________, and the lengths of ____________ and ____________ ____________ must be equal.

2. The length of the hypotenuse of the triangle on the left is ____________ and the hypotenuse of the triangle on the right is ____________.

3. The length of the leg of the triangle on the left is ____________ and the length of the leg of the triangle on the right is ____________.

Need

4. To solve the problem you need to find ____________.

Plan

5. What system of equations can you use to find the values of x and y?

6. What method(s) can you use to solve the system of equations?

____

____

7. What is the value of y? What is the value of x?

4-7 Congruence in Overlapping Triangles

Common Core State Standards
G-SRT.B.5 Use congruence ... criteria to solve problems and prove relationships in geometric figures.
MP 1, MP 3, MP 4

Mathematics Florida Standards
MAFS.912.G-SRT.2.5 Use congruence and similarity criteria to solve problems and prove relationships in geometric figures.
MP 1, MP 3, MP 4

Objectives To identify congruent overlapping triangles
To prove two triangles congruent using other congruent triangles

SOLVE IT! **Getting Ready!**

An assignment for your graphic design class is to make a colorful design using triangles. How many triangles are in your design? Explain how you count them.

MATHEMATICAL PRACTICES In the Solve It, you located individual triangles among a jumble of triangles. Some triangle relationships are difficult to see because the triangles overlap.

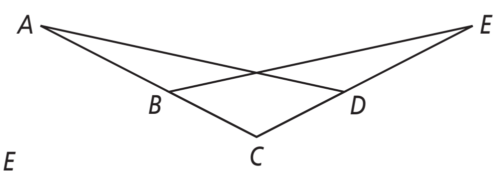
Essential Understanding You can sometimes use the congruent corresponding parts of one pair of congruent triangles to prove another pair of triangles congruent. This often involves overlapping triangles.

Overlapping triangles may have a common side or angle. You can simplify your work with overlapping triangles by separating and redrawing the triangles.

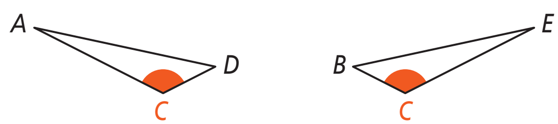
Problem 1 Identifying Common Parts

Think
How can you see an individual triangle in order to redraw it?
Use your finger to trace along the lines connecting the three vertices. Then cover up any untraced lines.

What common angle do $\triangle ACD$ and $\triangle ECB$ share?

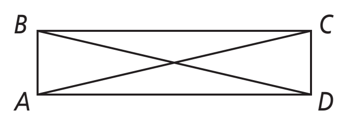


Separate and redraw $\triangle ACD$ and $\triangle ECB$.



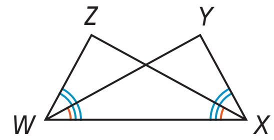
The common angle is $\angle C$.

Got It? **1. a.** What is the common side in $\triangle ABD$ and $\triangle DCA$?
b. What is the common side in $\triangle ABD$ and $\triangle BAC$?



Proof

Problem 2 Using Common Parts



Given: $\angle ZXW \cong \angle YWX$, $\angle ZWX \cong \angle YXW$

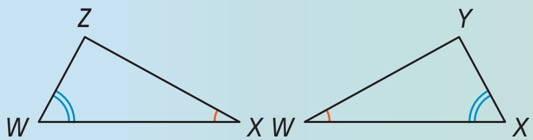
Prove: $\overline{ZW} \cong \overline{YX}$

Know

- $\angle ZXW \cong \angle YWX$ and $\angle ZWX \cong \angle YXW$
- The diagram shows that $\triangle ZWX$ and $\triangle YXW$ are overlapping triangles.

Need

A diagram of the triangles separated



Plan

Show $\triangle ZWX \cong \triangle YXW$. Then use corresponding parts of congruent triangles to prove $\overline{ZW} \cong \overline{YX}$.

$\angle ZXW \cong \angle YWX$
Given

$\overline{WX} \cong \overline{WX}$
Reflexive Prop. of $\cong$

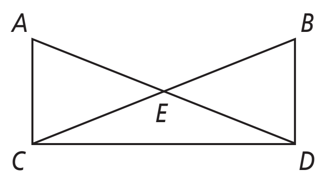
$\angle ZWX \cong \angle YXW$
Given

$\triangle ZWX \cong \triangle YXW$
ASA

$\overline{ZW} \cong \overline{YX}$
Corresp. parts of $\cong$ ▲ are $\cong$.

Got It? **2. Given:** $\triangle ACD \cong \triangle BDC$

Prove: $\overline{CE} \cong \overline{DE}$



Proof

Problem 3 Using Two Pairs of Triangles

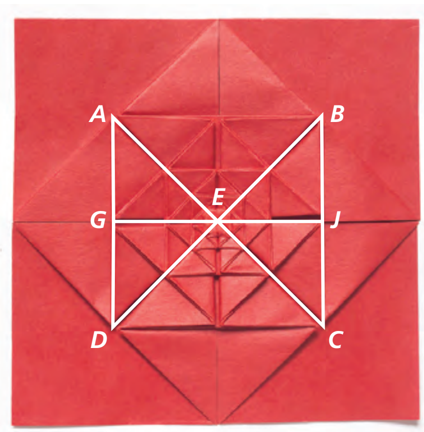
Plan

How do you choose another pair of triangles to help in your proof?

Look for triangles that share parts with $\triangle GED$ and $\triangle JEB$ and that you can prove congruent. In this case, first prove $\triangle AED \cong \triangle CEB$.

Given: In the origami design, E is the midpoint of $\overline{AC}$ and $\overline{DB}$.

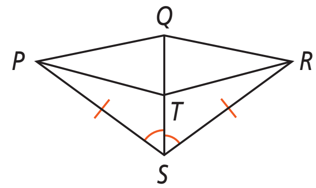
Prove: $\triangle GED \cong \triangle JEB$



Proof: E is the midpoint of $\overline{AC}$ and $\overline{DB}$, so $\overline{AE} \cong \overline{CE}$ and $\overline{DE} \cong \overline{BE}$. $\angle AED \cong \angle CEB$ because vertical angles are congruent. Therefore, $\triangle AED \cong \triangle CEB$ by SAS. $\angle D \cong \angle B$ because corresponding parts of congruent triangles are congruent. $\angle GED \cong \angle JEB$ because vertical angles are congruent. Therefore, $\triangle GED \cong \triangle JEB$ by ASA.

Got It? 3. Given: $\overline{PS} \cong \overline{RS}$, $\angle PSQ \cong \angle RSQ$

Prove: $\triangle QPT \cong \triangle QRT$



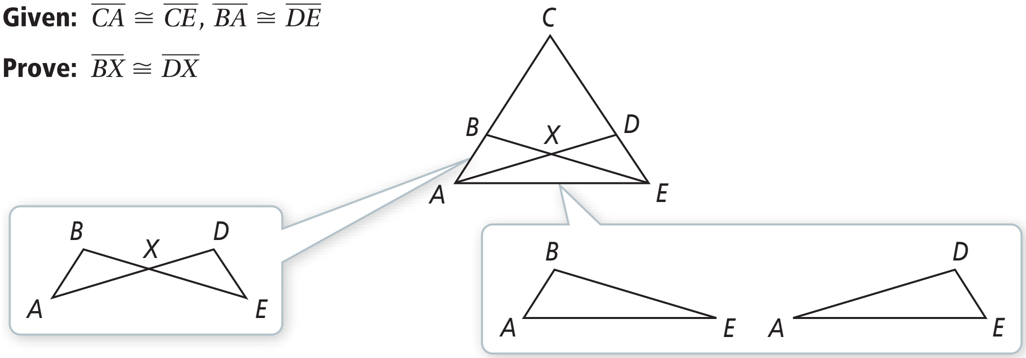
When several triangles overlap and you need to use one pair of congruent triangles to prove another pair congruent, you may find it helpful to draw a diagram of each pair of triangles.

Proof

Problem 4 Separating Overlapping Triangles

Given: $\overline{CA} \cong \overline{CE}$, $\overline{BA} \cong \overline{DE}$

Prove: $\overline{BX} \cong \overline{DX}$



Plan

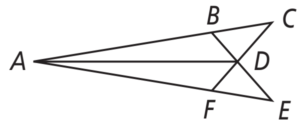
Which triangles are useful here?

If $\triangle BXA \cong \triangle DXE$, then $\overline{BX} \cong \overline{DX}$ because they are corresponding parts. If $\triangle BAE \cong \triangle DEA$, you will have enough information to show $\triangle BXA \cong \triangle DXE$.

Statements	Reasons
1) $\overline{BA} \cong \overline{DE}$	**1)** Given
2) $\overline{CA} \cong \overline{CE}$	**2)** Given
3) $\angle CAE \cong \angle CEA$	**3)** Base ∠s of an isosceles △ are ≅.
4) $\overline{AE} \cong \overline{AE}$	**4)** Reflexive Property of ≅
5) $\triangle BAE \cong \triangle DEA$	**5)** SAS
6) $\angle ABE \cong \angle EDA$	**6)** Corresp. parts of ≅ △s are ≅.
7) $\angle BXA \cong \angle DXE$	**7)** Vertical angles are ≅.
8) $\triangle BXA \cong \triangle DXE$	**8)** AAS
9) $\overline{BX} \cong \overline{DX}$	**9)** Corresp. parts of ≅ △s are ≅.

Got It? 4. Given: $\angle CAD \cong \angle EAD$, $\angle C \cong \angle E$

Prove: $\overline{BD} \cong \overline{FD}$

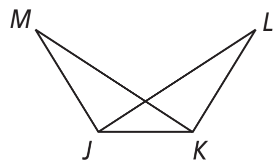


Lesson Check

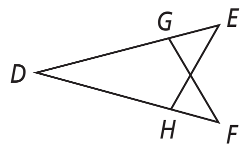
Do you know HOW?

Identify any common angles or sides.

1. $\triangle MKJ$ and $\triangle LJK$

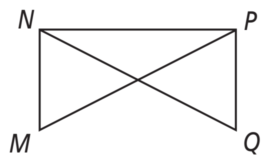


2. $\triangle DEH$ and $\triangle DFG$

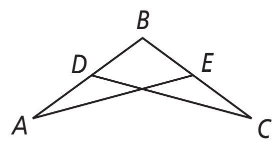


Separate and redraw the overlapping triangles. Label the vertices.

3.



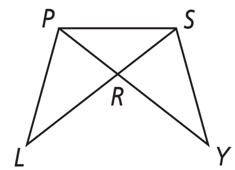
4.



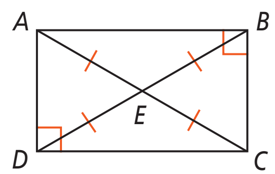
Do you UNDERSTAND? MATHEMATICAL PRACTICES

5. Reasoning In Exercise 1, both triangles have vertices J and K. Are $\angle J$ and $\angle K$ common angles for $\triangle MKJ$ and $\triangle LJK$? Explain.

6. Error Analysis In the diagram, $\triangle PSY \cong \triangle SPL$. Based on that fact, your friend claims that $\triangle PRL \ncong \triangle SRY$. Explain why your friend is incorrect.



7. In the figure below, which pair of triangles could you prove congruent first in order to prove that $\triangle ACD \cong \triangle CAB$? Explain.



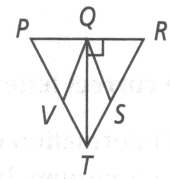
Name ______________________ Class ____________ Date ____________

4-7 Think About a Plan

Congruence in Overlapping Triangles

Given: $\overline{QT} \perp \overline{PR}$, $\overline{QT}$ bisects $\overline{PR}$,
$\overline{QT}$ bisects $\angle VQS$

Prove: $\overline{VQ} \cong \overline{SQ}$



Know

1. What information are you given? What else can you determine from the given information and the diagram?

____

____

2. To solve the problem, what will you need to prove?

____

Need

3. For which two triangles are $\overline{VQ}$ and $\overline{SQ}$ corresponding parts?

____

4. You need to use corresponding parts to prove the triangles from Exercise 3 congruent. Which two triangles should you prove congruent first, using the given information? Which theorem or postulate should you use?

____

5. Which corresponding parts should you then use to prove that the triangles in Exercise 3 are congruent?

____

Plan

6. Use the space below to write the proof.

4 Pull It All Together

Completing the Performance Task

Look back at your results from the Apply What You've Learned sections in Lessons 4-1, 4-3, and 4-4. Use the work you did to complete the following.

1. Solve the problem in the Task Description on page 217 by estimating the distance *XY* across the gorge. Show all your work and explain each step of your solution.

2. **Reflect** Choose one of the Mathematical Practices below and explain how you applied it in your work on the Performance Task.

   MP 1: Make sense of problems and persevere in solving them.

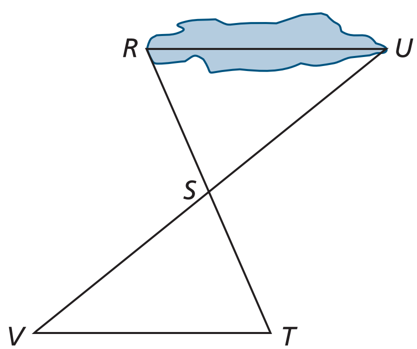
   MP 3: Construct viable arguments and critique the reasoning of others.

   MP 7: Look for and make use of structure.

On Your Own

Caitlin wants to estimate the distance across a pond. She begins at one end of the pond, shown as point *R* in the diagram below. She turns away from the pond, walks 300 ft in a straight line, and marks point *S*. She walks another 300 ft in the same direction and marks point *T*.

Next, Caitlin goes over to point *U* at the other end of the pond. She then measures the distance as she walks on a straight line to point *S* and finds that this distance is 340 ft. She continues another 340 ft in the same direction and marks point *V*.



a. Copy the diagram and label it with the given information.

b. What additional measurement should Caitlin determine to estimate the distance *RU* across the pond? Justify your answer.

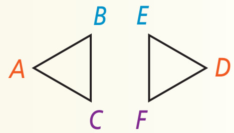
4 Chapter Review

Connecting BIG ideas and Answering the Essential Questions

1 Visualization
You can identify corresponding parts of congruent triangles by visualizing the figures placed on top of each other.

Congruent Figures (Lesson 4-1)

$\triangle ABC \cong \triangle DEF$

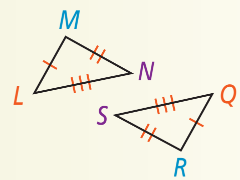


2 Reasoning and Proof
You can show two triangles are congruent by proving that certain relationships exist between three pairs of corresponding parts.

Proving Triangles Congruent (Lessons 4-2, 4-3, and 4-6)
Side-Side-Side (SSS), Side-Angle-Side (SAS), Angle-Side-Angle (ASA), Angle-Angle-Side (AAS), Hypotenuse-Leg (HL)

Using Corresponding Parts of Congruent Triangles (Lessons 4-4 and 4-7)

If $\triangle LMN \cong \triangle QRS$



$\overline{LM} \cong \overline{QR}$
$\overline{MN} \cong \overline{RS}$
$\overline{NL} \cong \overline{SQ}$
$\angle L \cong \angle Q$
$\angle M \cong \angle R$
$\angle N \cong \angle S$

3 Reasoning and Proof
You can tell whether a triangle is isosceles or equilateral by looking at the number of congruent angles or sides.

Isosceles and Equilateral Triangles (Lesson 4-5)
- The base angles of an isosceles triangle are congruent.
- All equilateral triangles are equiangular.
- All equiangular triangles are equilateral.

Chapter Vocabulary

- base angles of an isosceles triangle (p. 250)
- base of an isosceles triangle (p. 250)
- congruent polygons (p. 219)
- corollary (p. 252)
- hypotenuse (p. 258)
- legs of an isosceles triangle (p. 250)
- legs of a right triangle (p. 258)
- vertex angle of an isosceles triangle (p. 250)

Choose the correct term to complete each sentence.

1. The two congruent sides of an isosceles triangle are the _?_.

2. The side opposite the right angle of a right triangle is the _?_.

3. A _?_ to a theorem is a statement that follows immediately from the theorem.

4. _?_ have congruent corresponding parts.

4-1 Congruent Figures

Quick Review

Congruent polygons have congruent corresponding parts. When you name congruent polygons, always list corresponding vertices in the same order.

Example

***HIJK* ≅ *PQRS*. Write all possible congruence statements.**

The order of the parts in the congruence statement tells you which parts correspond.

Sides: $\overline{HI} \cong \overline{PQ}$, $\overline{IJ} \cong \overline{QR}$, $\overline{JK} \cong \overline{RS}$, $\overline{KH} \cong \overline{SP}$

Angles: $\angle H \cong \angle P$, $\angle I \cong \angle Q$, $\angle J \cong \angle R$, $\angle K \cong \angle S$

Exercises

***RSTUV* ≅ *KLMNO*. Complete the congruence statements.**

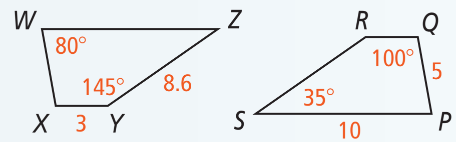
5. $\overline{TS} \cong$ ___?___

6. $\angle N \cong$ ___?___

7. $\overline{LM} \cong$ ___?___

8. $VUTSR \cong$ ___?___

***WXYZ* ≅ *PQRS*. Find each measure or length.**



9. $m\angle P$

10. QR

11. WX

12. $m\angle Z$

13. $m\angle X$

14. $m\angle R$

4-2 and 4-3 Triangle Congruence by SSS, SAS, ASA, and AAS

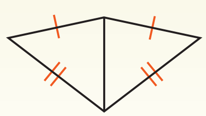
Quick Review

You can prove triangles congruent with limited information about their congruent sides and angles.

Postulate or Theorem	**You need**
Side-Side-Side (SSS)	three sides
Side-Angle-Side (SAS)	two sides and an included angle
Angle-Side-Angle (ASA)	two angles and an included side
Angle-Angle-Side (AAS)	two angles and a nonincluded side

Example

What postulate would you use to prove the triangles congruent?



You know that three sides are congruent. Use SSS.

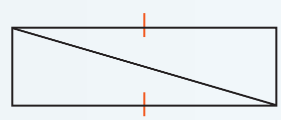
Exercises

15. In $\triangle HFD$, what angle is included between $\overline{DH}$ and $\overline{DF}$?

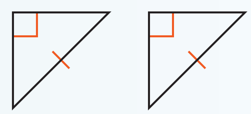
16. In $\triangle OMR$, what side is included between $\angle M$ and $\angle R$?

Which postulate or theorem, if any, could you use to prove the two triangles congruent? If there is not enough information to prove the triangles congruent, write *not enough information*.

17.



18.



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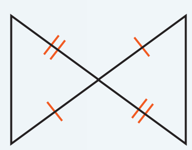

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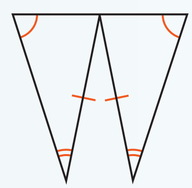


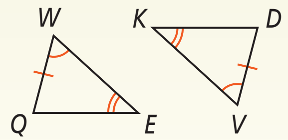
4-4 Using Corresponding Parts of Congruent Triangles

Quick Review

Once you know that triangles are congruent, you can make conclusions about corresponding sides and angles because, by definition, corresponding parts of congruent triangles are congruent. You can use congruent triangles in the proofs of many theorems.

Example

How can you use congruent triangles to prove $\angle Q \cong \angle D$?

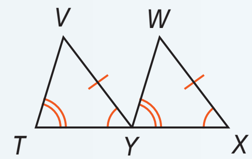


Since $\triangle QWE \cong \triangle DVK$ by AAS, you know that $\angle Q \cong \angle D$ because corresponding parts of congruent triangles are congruent.

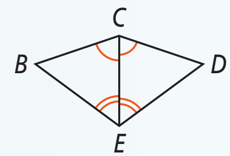
Exercises

How can you use congruent triangles to prove the statement true?

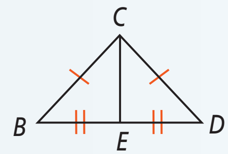
21. $\overline{TV} \cong \overline{YW}$



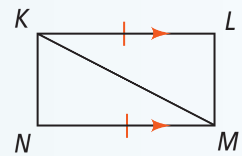
22. $\overline{BE} \cong \overline{DE}$



23. $\angle B \cong \angle D$



24. $\overline{KN} \cong \overline{ML}$



4-5 Isosceles and Equilateral Triangles

Quick Review

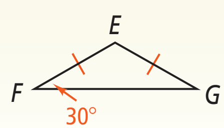
If two sides of a triangle are congruent, then the angles opposite those sides are also congruent by the **Isosceles Triangle Theorem.** If two angles of a triangle are congruent, then the sides opposite the angle are congruent by the **Converse of the Isosceles Triangle Theorem.**

Equilateral triangles are also equiangular.

Example

What is $m\angle G$?

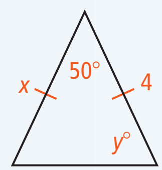
Since $\overline{EF} \cong \overline{EG}$, $\angle F \cong \angle G$ by the Isosceles Triangle Theorem. So $m\angle G = 30$.



Exercises

Algebra **Find the values of x and y.**

25.



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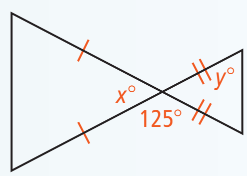

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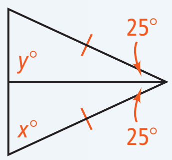

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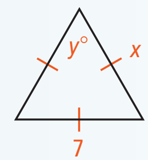


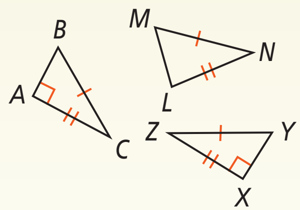
4-6 Congruence in Right Triangles

Quick Review

If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and a leg of another right triangle, then the triangles are congruent by the **Hypotenuse-Leg (HL) Theorem.**

Example

Which two triangles are congruent? Explain.



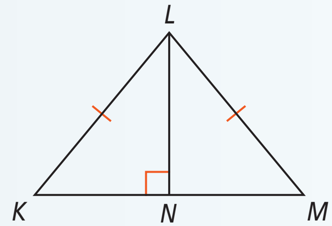
Since $\triangle ABC$ and $\triangle XYZ$ are right triangles with congruent legs, and $\overline{BC} \cong \overline{YZ}$, $\triangle ABC \cong \triangle XYZ$ by HL.

Exercises

Write a proof for each of the following.

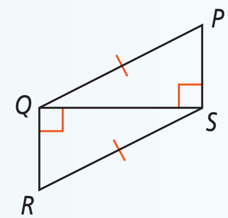
29. Given: $\overline{LN} \perp \overline{KM}$, $\overline{KL} \cong \overline{ML}$

Prove: $\triangle KLN \cong \triangle MLN$



30. Given: $\overline{PS} \perp \overline{SQ}$, $\overline{RQ} \perp \overline{QS}$, $\overline{PQ} \cong \overline{RS}$

Prove: $\triangle PSQ \cong \triangle RQS$



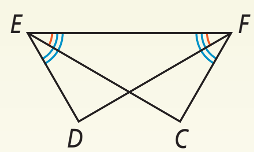
4-7 Congruence in Overlapping Triangles

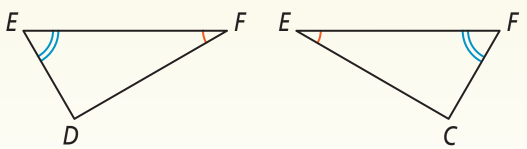
Quick Review

To prove overlapping triangles congruent, you look for the common or shared sides and angles.

Example

Separate and redraw the overlapping triangles. Label the vertices.

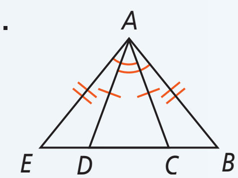




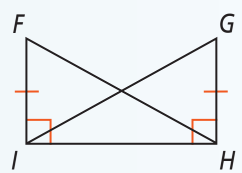
Exercises

Name a pair of overlapping congruent triangles in each diagram. State whether the triangles are congruent by SSS, SAS, ASA, AAS, or HL.

31.



32.



33.