THE KNOX SCHOOL

Calculus
2020 Summer Assignment

Directions: You must show all work, even for multiple choice. Any graphing problem should be done without a graphing calculator.

Due Date: First day of school! You will be held accountable for this material upon your return to school. Yes, that means a test or a quiz on this material is going to happen.

Multiple Choice

1. Susan won $2,000 and invested it into an account with an annual interest rate of 3.2%. If her investment were compounded monthly, which expression best represents the value of her investment after t years?

   1) $2000(1.003)^{12t}$

   2) $2000(1.032)^{\frac{t}{12}}$

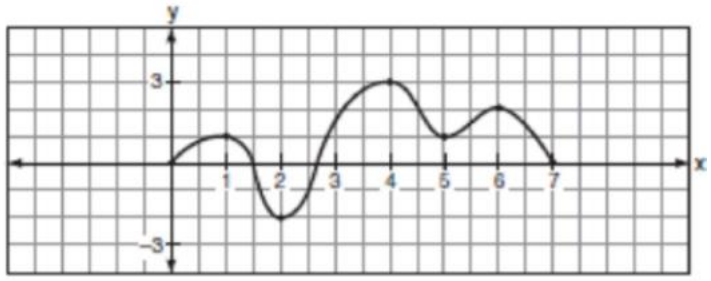
   3) $2064^{\frac{t}{12}}$

   4) $\dfrac{2000(1.032)^t}{12}$

2. If $f(x) = \frac{1}{2}x - 3$ and $g(x) = 2x + 5$, what is the value of $(g \circ f)(4)$?

   1) -13
   2) 3.5
   3) 3
   4) 6

3. The accompanying graph is a sketch of the functic
$y = f(x)$ over the interval $0 \le x \le 7$.



What is the value of $(f \circ f)(6)$?

1) 1
2) 2
3) 0
4) −2

4. What is the solution of the equation $2\log_4(5x) = 3$?

1) 6.4 2) 2.56 3) $\frac{9}{5}$ 4) $\frac{8}{5}$

5. A solution of the equation $2x^2 + 3x + 2 = 0$ is

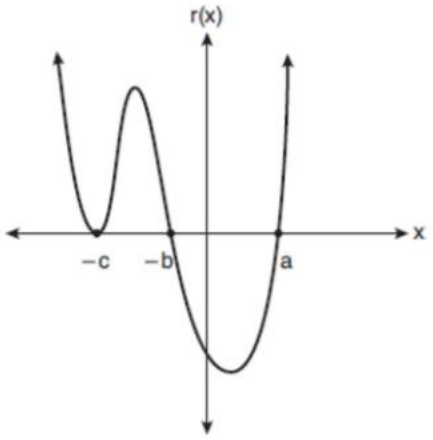
1) $-\frac{3}{4} + \frac{1}{4}i\sqrt{7}$

2) $-\frac{3}{4} + \frac{1}{4}i$

3) $-\frac{3}{4} + \frac{1}{4}\sqrt{7}$

4) $\frac{1}{2}$

6. A sketch of $r(x)$ is shown below.



An equation for $r(x)$ could be

1) $r(x) = (x-a)(x+b)(x+c)$
2) $r(x) = (x+a)(x-b)(x-c)^2$
3) $r(x) = (x+a)(x-b)(x-c)$
4) $r(x) = (x-a)(x+b)(x+c)^2$

7. Evan graphed a cubic function, $f(x) = ax^3 + bx^2 + cx + d$, and determined the roots of $f(x)$ to be ± 1 and 2. What is the value of b, if $a = 1$?

1) 1
2) 2
3) −1
4) −2

8. Which equation represents a graph that has a period of 4π?

1) $y = 3\sin\frac{1}{2}x$ 2) $y = 3\sin 2x$ 3) $y = 3\sin\frac{1}{4}x$ 4) $y = 3\sin 4x$

9. What are the amplitude and the period of the graph represented by the equation $y = -3\cos\frac{\theta}{3}$?

1) amplitude: −3; period: $\frac{\pi}{3}$
2) amplitude: −3; period: 6π
3) amplitude: 3; period: $\frac{\pi}{3}$
4) amplitude: 3; period: 6π

10. Identify the type of symmetry (if any) of the graph of the function.

$$g(x) = \frac{3x^2}{4x^2 + 1}$$

[A] origin symmetry

[B] x-axis symmetry

[C] y-axis symmetry

[D] no symmetry

Free Response Questions

11. Multiply and simplify.

$$\frac{a^3 - b^3}{3a^2 + 9ab + 6b^2} \cdot \frac{a^2 + 2ab + b^2}{a^2 - b^2}$$

12. Divide and simplify. State any restrictions on the variable.

$$\frac{x^2 + 3x - 28}{x^2 + 4x + 4} \div \frac{x^2 - 49}{x^2 - 5x - 14}$$

13. Simplify the following expression. Leave no negative exponents.

$$\left(\frac{x^6 y^{-3}}{27 y^{\frac{3}{5}}} \right)^{-\frac{1}{3}}$$

14. Simply the following radical.

$\sqrt[3]{-54x^7y^{11}}$

15. Solve for x. Leave answer as an exact value.

$$\left(e^x\right)^4 = e^{5x-6}$$

16. Given: $f(x) = 2x^2 + x - 3$ and $g(x) = x - 1$
Express $f(x) \bullet g(x) - [f(x) + g(x)]$ as a polynomial in standard form.

17. Solve for b.

$logb + \log(1 + b) = \log(32 - 3b)$

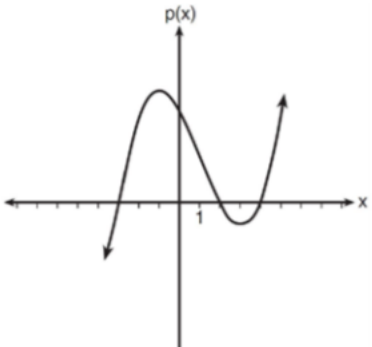
18. Solve the equation $4x^2 - 12x = 7$ algebraically for x.

19. Factor completely: $3ax^2 - 27a$

20. Factor completely: $5x^3 - 20x^2 - 60x$

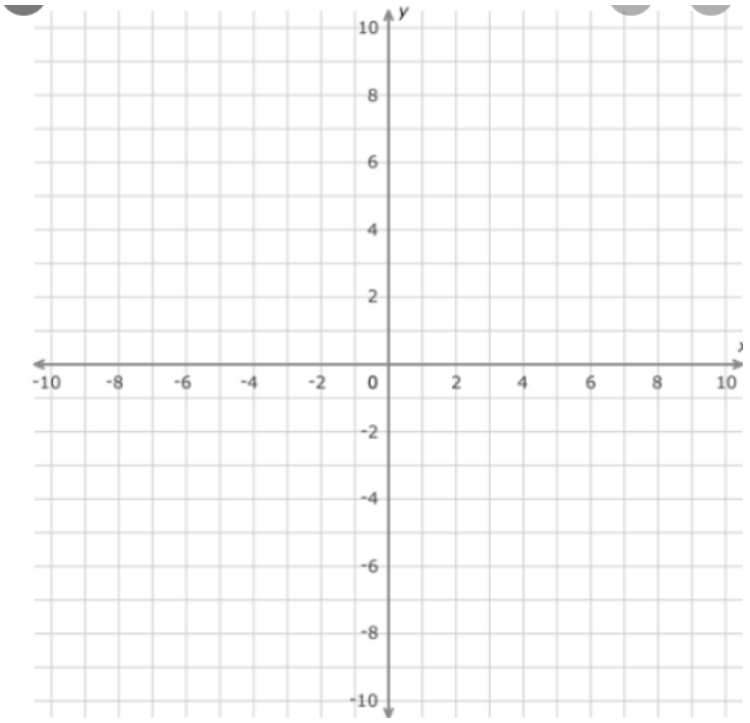
21. The zeros of the function $f(x) = 3x^2 - 3x - 6$ are

22. Based on the polynomial graph below, write a possible function, $f(x)$, in factored form that could represent the graph.

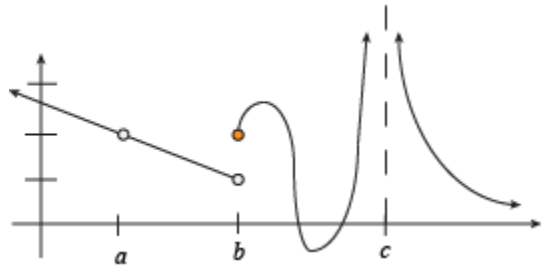


23. Graph the following piecewise function.

$$g(x) = \begin{cases} 5x, x \le 3 \\ x^2 + 4, x > 3 \end{cases}$$



24. A piecewise function is given below. State the x-values of all discontinuities and the type of discontinuity: hole, jump, vertical asymptote. Also state if the discontinuity is removable or non-removable.



25. Graph at least one cycle of the following trig function.

$y = -3\sin(x - \pi) + 1$

26. Solve by rewriting both sides using the same base.

$$3^{x-1} = \left(\frac{1}{243}\right)^{x+4}$$

27. Algebraically show if the function is even, odd, or neither.

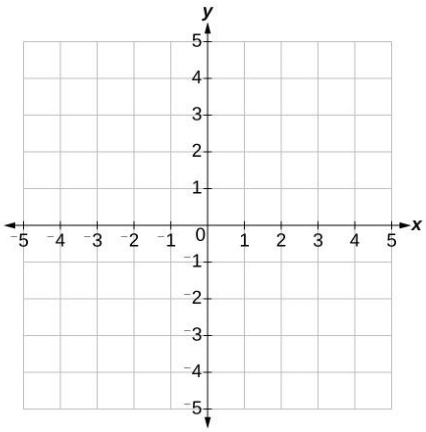
$f(x) = 4x^3 + 5x - 1$

28. Find 2 values for θ on the interval $[0,2\pi]$ that satisfy the equation:

$$cos\theta = -\frac{\sqrt{3}}{2}$$

29. Graph the rational function below. State the equations of any asymptotes, the coordinates or any holes. If the function has a horizontal asymptote, be sure to check to see if it gets crossed.

$f(x) = \frac{x+3}{x^2+2x-3}$



30. The inverse of the function $f(x) = \frac{x+1}{x-2}$ is

31. Find the exact value of the trig functions below.

a. $\sin\left(\frac{7\pi}{6}\right)$

b. $\cos\left(\frac{3\pi}{4}\right)$

c. $tan\left(\frac{4\pi}{3}\right)$

d. $\csc\left(\frac{2\pi}{3}\right)$

32. Graph one period of the cosine function given below.

$$y = 2\cos\left(\frac{1}{3}x\right) - 4$$

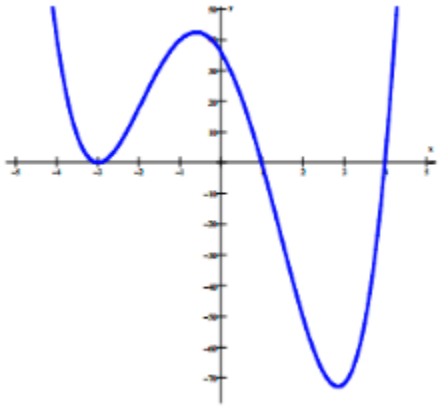
33. Use long division to divide the following polynomials.

$$\frac{3x^3 - 4x^2 + 5x - 5}{x - 2}$$

34. Find the equation of any vertical, horizontal, or slant (oblique) asymptotes.

$$f(x) = \frac{2x - 1}{x^2 - 3x - 4}$$

35. Use the graph of the polynomial function below to answer the following questions. Justify your answer.



A. Is the degree of the polynomial even or odd?

B. Is the leading coefficient positive or negative?

C. Is the function even, odd, or neither?

D. Why is $(x + 3)^2$ a factor of the polynomial?

E. What is the minimum degree of the polynomial?

F. Give one equation that could represent the function in the graph above.

36. Use the law of cosines to find the missing side of the triangle below.